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Codify a trihedral inventory control model in terms of inflation in state of non-allowed shortage for an incorruptible commodity

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ABSTRACT
this research seeks to provide a mathematical model for a trihedral inventory control system. This three-level chain, including manufacturing, warehouse and seller. In situations where a specific product is produced under inflation and the impact of inflation with exponential function is on the price of units. also in this model demand rate, production rate and Lead Time are constant. In this model, shortage is non-allowed. With these hypotheses obtains total cost function and determine limitations each of the decision variables then determine optimal order point by using MATLAB software .
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Keyword:
Inventory control, trihedral, Non-Allowed shortage, order economic size, inflation, supply chain, MATLAB software

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INTRODUCTION

After presenting the first model of inventory control in the early twentieth century which were investigated in very primitive case, changes were made on it from various aspects, Which was to expand and adapt to actual conditions. Also according to some conditions in various issues, these models will be affected by certain parameters and limitations. One of effect has been studied on the models is the impact of inflation. in 1964 Hadley consider the time value of money, He calculated order size with average annual cost method and discounted costs and proved that almost cost obtained from both methods are equal. (Hadley, 1964) Bazakt in 1975 was the first person who raised Inventory control issues with regard to inflation(Bazakt, 1975). In 1977, offered a model of economic order in which discount had been considered (Bierman and Thomson, 1977). In 1997, also examined economic order model with variable demand (Ray and Chad Hori, 1997). In 2015 a development method was done to calculate the optimal mode in multi-parametric mode for economic order quantity model (Salvatore et al., 2015). after that amount of inventory cost affected by the inflation has been assumed in the form of a single seller and single buyer Supply Chain in the form of a EOQ model and has been resolved in both cases which lead time has normal distribution and free distribution(Jindal, 2016). Damyad provide a trihedral model seller, warehouse and manufacturing under the inflation and investigate for corruptible and incorruption commodities in cases of allowed and non-allowed shortage (Damyad, 2016). Here we examine the inventory model when the commodity is incorruption and there is no shortage in situations where there is a inflation and solve and developed mathematical model of this situation.

2. Research model

In this research seeks to find the optimal model of a trihedral system which are shown in Figure 1**.** As is clear from Figure. specific amount of a commodity is produced in level of production then the commodity is backloged in the warehouse. amount of specified stored commodity is sent to level of seller that discussed Describe the process in the following and will obtain level of equations.

figure 1

In Table 1 introduces used symbols in this research which will be used of it in describing the model and relevant equations.

Table 1: introduction Symbols

Definition	Symbol	Row	Definition	Symbol	Row
Unit cost of production product at the beginning of the first cycle	u	14	*Total Holding Cost*	THC	1
The cost of product order in the first cycle of seller level	C_v	15	*Total Ordering Cost*	TOC	2
The unit cost of product orders in the first cycle level warehouse	C_s	16	*The time interval from reaching order in each cycle until re-order seller*	t^*	3
inflation rate	$\dot{\delta}$	17	*The cost of holding at the beginning of the first cycle of seller level*	h_v	4
Interest rate	$\ddot{\delta}$	18	*The cost of product manufacturing in the i cycle*	MC_i	5
lead time Product (constant number)	L	19	*The cost of product holding in the i cycle*	HC_i	6
The unit cost of launching in production level in the first cycle	C_{se}	20	*Amount of inventory in the cycle i*	H	7
Interest rate of net of inflation $(\delta = \dot{\delta} - \ddot{\delta})$	δ	21	*Reorder point (decision variables)*	r	8
lead time Product (constant number)	L	22	*amount of each Order (decision variables)*	Q	9
The total cycles at the level of seller within a year	N	23	*The number of seller cycles are within a production cycle (decision variables)*	m	10
The total number of cycles at the level of warehouse for one year $n = \frac{N}{m}$	n	24	*Demand Product rate (constant number)*	D	11
Time of a cycle at the level of seller	T	25	*Production rate (constant number and P> D)*	P	12

Definition	Symbol	Row	Definition	Symbol	Row
Time	t	26	*When in production cycle (its Time is m times of a seller cycle) product line works*	$\bar{t}$	13
Amount of Safety stock	r_0	27			

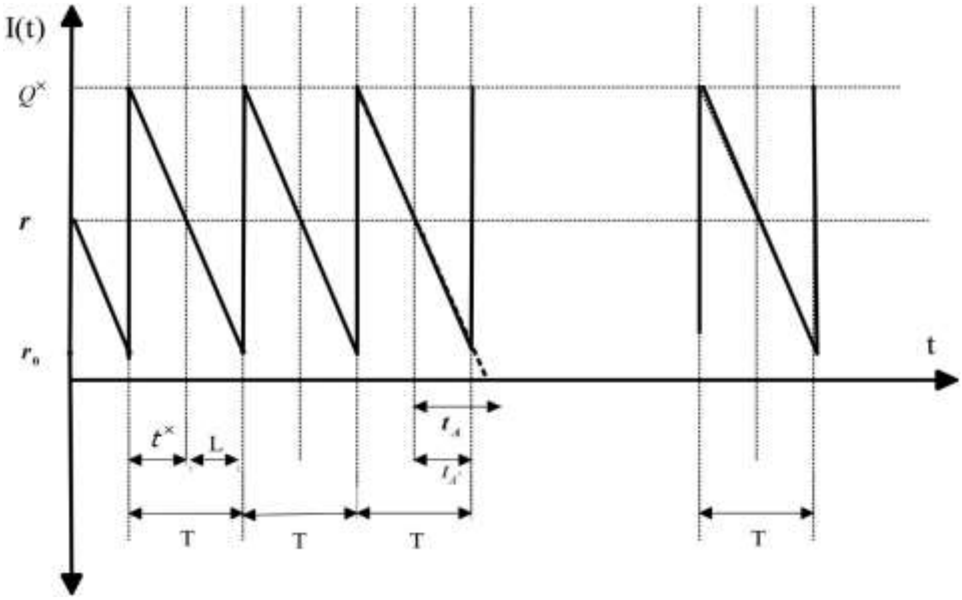
Also research hypotheses is described in Table 2.

Hypotheses
1. *Demand is always a constant number D in all cycles.*
2. *production rate is always constant number P and always is more than rate of demand. (P> D)*
3. *Model is for a Single-product process.*
4. *Commodity isn't Corruptible.*
5. *shortage Is not- allowed*
6. *Lead Time Delivery is constant*
7. *During time horizon is one year*
8. *The cost of Faced with shortages maintenance, purchasing and order each of them are as a function of the rate of inflation in each period.*
9. *Time value of money and interest rate and inflation has been considered With fixed rate.*
10. *Model has been composed trihedral of manufacturer, warehouse and seller*

Table 2: Hypotheses

2.1. seller Level

At this Level of amount of inventory is declining influences the demand rate until amount of balance reaches to order point and at the same time, order command will be issued to amount of Q (R is reorder point). But these order is collected in L units of time after it. At this moment, which starts first cycle amount of inventory under the influence of demand reduced with a fixed rate D of the total inventory.Until the moment (t_A) amount of inventory reaches zero. After these moment is not able to meet the demand. So sure should receipt commodity until this moment that if does not reach there will be shortage that this is contrary to the assumption so we will assume that commodity receipt at the t_A time that this time is always smaller than t_A . it is necessary to mention at the $t_{\acute{A}}$ moment, amount of inventory reaches to r_0 amount . At this moment, again certain amount of inventory order sent into this level and this process is repeated. That this process has been shown in Figure 1.



To analyze process of inventory and access to relevant equations , initially examine inventory behavior in time for level of seller and we know that inventory at this level, only is affected amount of demand that it is also constant number. So inventory rate in time can be obtained from following equation.

$$\frac{dI_1(t)}{dt} = -D$$

By solving mentioned differential equation will have inventory function in below form. (Paryab, 2007)

$$I_1(t) = I_0 - D\,t$$

The amount of commodity beginning of each cycle is obtained through the following:

$$I_0 = Q^* = Q + r_0$$

However,by Identify the inventory behavior, at the time, Should calculate the inventory cost for this level. Before addressing this issue , calculates certain times in the model that all of them is obtained by considering the boundary conditions on the inventory equation and time in seller level .

$$t_A = \frac{I(0)}{D} = \frac{r}{D}$$

$$t_{\acute{A}} = \frac{I(0)}{D} = \frac{r - r_0}{D}$$

$$t^* = \frac{Q^* - r}{D} = \frac{Q + r_0 - r}{D}$$

also time duration of a cycle in seller level can be calculated from the following relationship.

$$T = L + t^* = t_{\acute{A}} + t^* = \frac{Q}{D}$$

A. Holding Costs

To obtain holding cost should have amount of inventory along with holding time in each cycle, Initially earns amount of inventory in the first cycle ,also know that in all cycles amount of inventory is identical .

$$H_v = \int_0^{t^* + t_A} I_1(t)\,dt = (Q + r_0)(T) - \frac{1}{2}D(T)^2 = r_0 * T + \frac{1}{2}D(T)^2 = \frac{Q}{D}\frac{Q}{2}$$

It is clear that the holding cost of level is equal to the holding cost of total cycle

$$THC_v = HC_1 + HC_2 + \cdots + HC_n$$

On the other hand amount of inventory is constant in each cycle and time of each cycle, unit cost is just holding that changes under inflation and the time value of money and this price change is with the following function on the basis of time.

$$h_v\left(e^{\delta(T)}\right)$$

Now, calculate the rest of cycles with puting first cycle holding cost as the value of cost base.

$$H_v h_v\left(1 + e^{\delta(T)} + e^{2T\delta} + \cdots + e^{(N-1)T\delta}\right)$$

Can see that the value of in parentheses is a geometric sequence .So for the holding cost of entire cycles of seller we will have:

$$= H_v h_v \left(\frac{1 - e^{NT\delta}}{1 - e^{T\delta}}\right)$$

B. Ordering Costs

Ordering costs is considered at the beginning of the cycle that changes according to inflation in each cycle we assume ordering costs of the first cycle $C_v = C_{v1}$ is obvious that ordering costs in the next cycle is updated with function of inflation rate and the value of money.

So amount of total ordering cost is as follows:

$$TOC = OC_{v1} + OC_{v2} + \cdots + OC_{vn}$$
$$= C_v\left(1 + e^{\delta(T)} + e^{2T\delta} + \cdots + e^{(N-1)T\delta}\right)$$
$$= C_v \left(\frac{1 - e^{NT\delta}}{1 - e^{T\delta}}\right)$$

According to the above was mentioned except inventory costs could be gained the total cost of inventory in the state inflation at the level of Seller.

$$TC_v(Q, r, N) = THC + TOC$$
$$= \left(\frac{Q}{D}\left(\frac{Q}{2} + r_0\right)h_v + C_v\right)\left(\frac{1 - e^{NT\delta}}{1 - e^{T\delta}}\right)$$

2.2. warehouse Level

At this level, inventory is influenced the decline of amount of commodity by delivering to seller level in each cycle of seller and also increases the commodity in any production cycle. As has been shown in Figure 2. At the beginning of each cycle mQ amount of produced commodity in production level immediately and without delay is enterd to warehouse level. At first send Q value of it to seller level. after one cycle seller T amount of Q value of the inventory again is sent to seller level this process continues until that inventory amount of this level reaches zero after m times send the commodity. Warehouse cycle (mT) ends with resend.

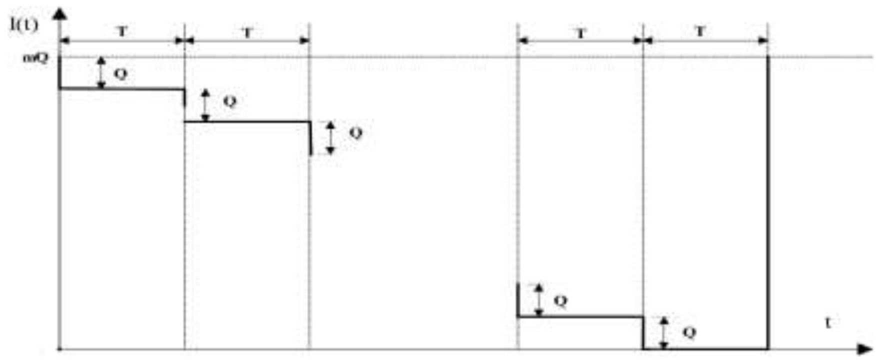


Chart 2 inventory in interfaces Warehouse level

For analysis inventory at this level, know that inventory rate does not change with respect to time only in moments of time is subject to change, therefore, inventory rate equation is as follows.

$$\frac{dI_2}{dt} = 0$$

By using above equation, obtain costs of this level through following.

A.Holding Costs

To determine the holding costs should be identified amount of commodity in holding time. To achieve this goal in Warehouse cycle, amount of available commodity is mQ at the beginning of each cycle. After every T units of time is reduced to amount of Q of Commodity. So amount of commodity in time obeys according to the following equation.

Therefore, the inventory - time commodity in a warehouse cycle is as follows

$$H_s = H_{s1} = (m-1)QT + (m-2)QT + \cdots + 0$$

this equation is a arithmetic sequence, So it will be as follows.

$$H_s = H_{s1} = \frac{m(m-1)QT}{2} = \frac{m(m-1)Q^2}{2D}$$

In the case of existence of inflation, the unit cost of holding of the cycle changes to another cycle. So after adjusting for inflation, the total holding cost at the warehouse level will be is as follows

$$\int_0^T (I_2(0) - Q)\ dt = (I_2(0) - Q)\ T$$

$$THC_s = \frac{m(m-1)QT}{2} h_s \left(1 + e^{mT\delta} + e^{2mT\delta} + \cdots + e^{(n-1)mT\delta}\right)$$

$$= \frac{1}{2D} m\ (m-1)\ Q^2 h_s \left(\frac{1 - e^{nmT\delta}}{1 - e^{mT\delta}}\right)$$

B. Ordering Costs

If for any time of sending commodity from manufacturing to warehouse costs are paid at the rate unit C_s. Then amount of total ordering cost will be as follows

$$TOC_s = C_s\left(1 + e^{mT\delta} + e^{2mT\delta} + \cdots + e^{(n-1)mT\delta}\right) = C_s\left(\frac{1 - e^{nmT\delta}}{1 - e^{mT\delta}}\right)$$

According to the above calculations in amount of total cost of inventory at interface level warehouse is as follows

$$TIC_s(Q,r,m,n) = THC + TOC$$

$$\left(\frac{1}{2}m(m-1)QTh_s + C_s\right)\left(\frac{1 - e^{nmT\delta}}{1 - e^{mT\delta}}\right)$$

2.3. Manufacturing level

in level of manufacturing of commodity at a time when the car started to produce will be added to amount of inventory and in time to stop all commodity produced will be sent to the warehouse. So that commodity does not remain at the manufacturing level. If production be at fixed rate $P\ (P \geq D)$ and a cycle of this level is considered mT. at the moment of t_s machine Start and started to produce and the machine is turned off at the moment of mT and all inventory is transferred to the warehouse. This process has been shown in Figure -3.

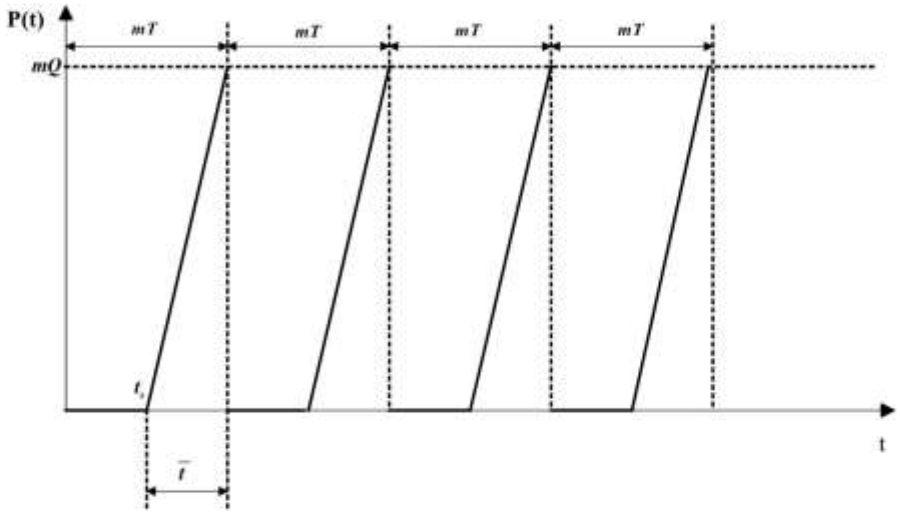


Figure 3. Diagram of inventory time in the level of manufacturing

According to the above figure Can be knew existing equation in the manufacturing cycle influenced by production rate. so according what to stated about inventory flow will have:

$$\frac{dI_3}{dt} = P$$

$$I_3(t) = Pt$$

By having inventory relations in this level know that at the moment of t_s amount of inventory is zero. So with this issue and at the mT moment , amount of inventory reaches mQ can calculate the amount of t_s time .

$$P\,\bar{t} = mQ$$

A. Set-up Costs

During Set-up the machine amount of costs should be spent and because we are in inflation space. This is made price change in each cycle so will have:

$$\text{TSeC} = C_{se}\left(1 + e^{mT\delta} + e^{2mT\delta} + \cdots + e^{(n-1)mT\delta}\right) = C_{se}\left(\frac{1 - e^{nmT\delta}}{1 - e^{mT\delta}}\right)$$

B. Production costs

To produce any Commodity, Production costs have specific amount. We know that in mechanical production process activities for $\bar{t}$. Therefore, the total production is obtained as follows with the symbol TP.

$$TP = P(\bar{t})$$

In the case that there will be inflation, the cost of production in all the cycles is as follows

$$TPeC = m\,Q\,u\left(1 + e^{mT\delta} + e^{2mT\delta} + \cdots + e^{(n-1)mT\delta}\right) = m\,Q\,u\left(\frac{1 - e^{nmT\delta}}{1 - e^{mT\delta}}\right)$$

C.Holding Costs

As was mentioned, manufactured commodities will be sent to the warehouse at the end of production cycle. But in a period of production, amount of commodity is also kept at this level that should also determine this amount and their costs. Amount of inventory - time a production cycle will be calculated by the following equation.

$$H = \int_0^{\bar{t}} I_2(t)dt = \int_0^{\bar{t}} Pt\,dt = \frac{m^2Q^2}{2P}$$

By taking into account the impact of inflation and the amount of time - inventory are equal together in all cycles, The holding costs in production cycle is as follows.

$$THC_p = \frac{m^2Q^2}{2P}\left(\frac{1 - e^{nmT\delta}}{1 - e^{mT\delta}}\right)h_p$$

According to the above calculations, total inventory cost in production levels by adjusting for inflation will be as follows.

$$TIC_p(m,n,T,Q) = TSeC_p + TPeC_p + THC_p$$

$$\left(C_{se} + m\,Q\,u + \frac{m^2Q^2}{2P}h_p\right)\left(\frac{1 - e^{nmT\delta}}{1 - e^{mT\delta}}\right)$$

So total inventory cost in the trihedral system is as follows:

$$TIC(Q,r,N,m) = TIC_v(Q,r,N) + TIC_s(Q,r,N,m) + TIC$$
$$= \left(\frac{Q}{D}\left(\frac{Q}{2} + r_0\right)h_v + C_v\right)\left(\frac{1 - e^{NT\delta}}{1 - e^{T\delta}}\right) + \left(\frac{1}{2D}m(m-1)\,Q^2\right.$$
$$\left. + \frac{m^2Q^2}{2P}h_p\right)\left(\frac{1 - e^{nmT\delta}}{1 - e^{mT\delta}}\right)$$

2.4 Limitations

According to figures provided at different levels of model and also conditions referred to in them in each level, has a series of limitations that are expressed in below in breakdown of each level.

A. limitation period of one year

As is evident, the number of cycles of level seller is equal to N and the duration of each cycle is T and cycles of warehouse and manufacturing level n cycle is implemented in a year and each cycle time is mT So will have:

$$NT = mnT = 1 \quad \rightarrow \quad T = \frac{1}{N} = \frac{1}{mn}$$

B. limitation related to the period from order point to reaching commodity (t_A) in seller cycle

The time interval between point order and safety stock point (rereceipt point) in each cycle is certain value.

$$T = L + t^* = L + \frac{Q + r_0 - r}{D} = \frac{Q}{D} \quad \rightarrow \quad r - r_0 = DL$$

C. limitation related to the period from order point to reaching commodity (t_A) in seller cycles out of inventory point (t_A) in seller cycle

Distance from the order point to out of inventory point, Should be at least as much lead time because if this value be less , occurs shortage and shortages in supposition of the problem is not allowed and because according to the supposition plan is for a period of one year. The maximum to this time is one year so we will have:

$$L \le t_A \le 1$$

$$t_A = \frac{r}{D} \quad \rightarrow \quad LD \le r \le D$$

By applying prior restraint will have:

$$0 \le r_0 \le (1 - L)D$$

C. limitation related to the period from entering order up to reaching to order point (t^*) in seller cycle

As previously was mentioned, time of a cycle seller follows of $T = t^* + L$ function. So will have.

$$0 < t^* < 1 - L$$

$$Q_{max} = (1 - L)\,D + r - r_0$$

$$Q_{min} = r - r_0$$

E. Limit the number of seller cycle in warehouse cycle

According to the equality time of planning at different levels that is for one year according to following relation, amount of maximum warehouse and manufacturing cycle time creates following limitations for us.

$$nmT = NT \rightarrow m = \frac{1}{nT} \qquad 1 \le m \le \frac{1}{T} \rightarrow 1 \le m \le \frac{D}{Q}$$

F: limitation related to production time in each cycle

production cycle includes a time period machine uptime and a machine rest time. Maximum machine uptime if can be achieved that machine rest time be zero so this limits is as follows:

$$\bar{t} < mT$$

$$\bar{t} = \frac{mQ}{P}$$

$$\frac{mQ}{P} < mT \rightarrow \frac{Q}{T} < P \rightarrow \frac{Q}{\frac{Q}{D}} < P \rightarrow D < P$$

Basic premise of model based on be greater than production rates on demand rate this limitation has already been considered and is additional.

G: Conclusion of model general limitation

From explanation was previously mentioned , Limitations of model is altogether as following form.

$$0 \le r_0 \le (1 - L)D$$

$$DL \le Q \le D$$

$$1 \le m \le \frac{D}{Q}$$

3. Solving method of model

It can be seen that the solution of non-linear function obtained from classic method is very complex and practically impossible. To solve this functionshould use MATLAB software. Know that values of three decision variable in the objective function $(Q, r_0, m)$are natural number. So by using mentioned software, implemented all possible states for them under mentioned limitations and determine the optimal point.

It should be noted that in the following table, determine parameter values in issue in numerical solution .

	C_s	C_v	C_{se}	u	D	P
	1000	1200	3500	500	1000	5000
h_v	h_s	h_p	δ	$\dot{\delta}$	$\ddot{\delta}$	L
20	30	25	-0.05	0.15	0.2	$5/365$

Table 2: Numerical values of parameters

By running the application in software and obtain optimal values based on control of all possible values is as follows.

TIC	m	r_0	r	Q
511225	**1**	**0**	**DL**	**474**

Table 3: software Output

4. Conclusion

About reviewed numerically, It can be seen that safety stock point has been suggested zero in all cases, and algorithm always offers amount of this, As well as whatever holding costs are larger against the costs order , reduced order quantity, It is clear that whatever amount of holding of commodity to be more advantageous. m value desire to at least more because the production and seller cycle time are equal. In the following table show the influence of some of them in decision variables and the objective function .

TIC	r_0	Q	m	D	P	u	L	δ	C_{se}	C_v	C_s	h_p	h_s	h_v	row
511225	0	474	1	1000	5000	500	$\frac{5}{365}$	-0.05	3500	1200	1000	25	30	20	1
620743	0	84	1	"	"	"	"	"	"	"	"	3000	2000	1000	2
637371	0	137	1	"	"	"	$\frac{50}{365}$	"	"	"	"	"	"	"	3
795426	0	22	3	"	"	"	$\frac{5}{365}$	"	"	"	"	2000	2000	6000	4

Table 4: Sensitivity Analysis of Parameters

The obtained model in terms of inflation for a trihedral chain offer to us amount of optimal ordering and optimal point of order, however, can expande this model by changing in the model adapted to different conditions and added domain of using it.

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