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A mathematical model for P-hub median location problem to multiple assignment between non-hub to hub nodes under fuzzy environment

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ABSTRACT

One group of the optimization problems is Hub location problem. Hub facilities are used in transportation network which are provided transfer of products, information, passengers and postal package through of intermediate node instead of direct transfer. One main group of these problems is discrete hub median location that seeks to minimize total transportation cost. In this paper, we present hub median location problem with multiple allocation of non-hub nodes to hub nodes, moreover it is possible to connect non-hub nodes directly with considering a penalty coefficient. The numerical example for CAB25data set -which is related to flow and distance between 25 cities in America- with establishment cost under fuzzy circumstance solved and presented.

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1. Introduction

Network design is one of the important problems in telecommunication and transportation problems which have a prodigious effect in utilization and economic saving. Over the past two decades, location and hub allocation problems take into consideration by researchers, among network design problems. This problem is formulated by O'Kelly ¬[1] for the first time. Most applications of these problems may cause to develop a large bearing on the early model according to the actual conditions. It is possible to prevent direct connection costs by the usage of hub. Hub location problems consist of finding optimal number of hubs, optimal situation of hubs, and optimal allocation of network nodes to hubs. Hub, in term of lexical means the spoke in the middle of wheels. Campbell (2007) [2] defines hub in the location problems as: "facilities that serve to origins and destinations as transfer and exchange points for transportation and telecommunication systems." Hub location problems have many applications for transportation and telecommunication networks. With the rapid growth of the air express delivery of postal packages, hub location problems have helped to improve these systems. Hub location network is widely used in telecommunication systems. These problems have various applications such as: postal delivery services, air services (telephone network, video conferencing, computer communication, and mobile network), emergency services, and logistics systems.

Campbell divided hub location problems in four categories based on objective functions:

- P-hub median location problem
- P-hub center location problem
- Fixed cost hub location problem
- Hub covering problem

I-A. P-hub Median Problem

In this model, the number of p-hub is determined and the objective of this model is minimizing total cost (time, distance, etc.) of transportation so that there is a connection between each pair of n nodes. P-hub median problem is a NP-hard problem in single and multiple allocation even though the situation of hubs are determined.

I-B. P-hub Center Problem

The objective of this model is minimizing the maximum cost (travel time, distance, etc) so that should be a connection between each pair of n node. This problem is a kind of MINIMAX problem and it was modeled as a single and multiple allocation models by Campbell for the first time. He defined three different kinds of problems according to the objective functions:

1. Minimizing the maximum connection cost between each pair of origin-destination nodes that is used in perishable distribution systems or time sensitive systems.

2. Minimizing the maximum connection cost in each connection routes (origin- destination, hub-to-hub, hub-destination). For example, products that need to heat, cold, etc.
3. Minimizing the maximum connection cost between hubs and origins and between hubs and destinations: in some problems that the transition between the hubs is not fitted on cost.

Campbell introduced a quadratic formulation for the models and made them linear with additional variables. Kara and Tanse proposed linear programming for all models in single allocation mode. They showed that their models are more efficient than Campbell`s models. They also provided a combinatorial formulation of these problem, and proved that it is difficult. Ernst presented a new and more efficient model for this problem in single allocation mode. He defined rk as a maximum cost between hub k and allocated nodes to hub k. the number of constrains in his model is less than Kara and Tanse`s model. He proposed an integer solving method and some heuristic methods for solving his model. They showed their efficiencies with experiment on CAP and AP benchmark problem. Gaverioolik proposed a congestion technique for Ernst model. Alumur et.al used the fuzzy algorithm for solving Ernst model in single allocation mode.

I-C. Hub Location with Fixed Cost

In this model minimizing the total cost (time, distance, or etc) to transport is considered. a fixed cost for establishment of the hub or the arc with the nature of the problem is considered. In general, this model simultaneously examine three objectives:

a) Optimal number of hubs
b) Optimal location of hubs
c) Optimal allocation of non-hub to hub nodes

O'Kelly presented a new quadratic integer formulation of single allocation p-hub median problem. He used two heuristic algorithms for solving his models and examine these on CAB data. Campbell presented the first integer formulation for USAPHLP. He considered the minimum threshold flow for each arc. Schneur et.al (1996) presented a new mixed integer programming for this problem. Krishnamoorthy (1996) [3] proposed another integer linear programming which is used fewer variables and constrains than previous models. They showed the ability of their models on Australian post data. Ebery (2001) [4] presented another formulation for USAPHLP that has a fewer variables and constrains than previous models. O'Kelly proposed a two stages method for single allocation hub location problem (USAHLP) that is included a good estimate of the upper bound based on three heuristic answers and calculation of the lower bound. Sue Abdinnour-Helm et.al [5] used a Genetic Algorithm for solving large instance and branch and bound for small instance of this problem. Sue Abdinnour-Helm proposed a hybrid genetic and tabu search algorithm for solving the problem so that genetic algorithm finds the number and location of hubs and uses tabu search for finding the optimal allocation of non-hub nodes to hubs. Topcuoglua [6] presented a heuristic genetic algorithm for estimating the number and the location of of hubs and the optimal allocations in USAHLP. He showed his algorithm is more efficient than Abdinnour-Helm algorithm. Chen [7] used the upper bound for determining the number of hubs and SA and TS for solving USAHLP and showed the efficiency of his algorithm.

In this paper, we study median hub location problem that the number of hub nodes is unclear and it is determined by solving the problem. Another difference of this model with the general model is in possibility of creating rout between two non-hub nodes. In reality, there is a direct connection when the cost of route that passes through the hub node is greater than direct route. This situation make our model more flexible than other models. In this model other costs are considered for closing the model to real world and choosing better choices than the past. In the next section we describe the model.

2. The Models

This paper introduces multiple hub median problem which the direct connections between non-hub nodes are allowed. There are three kind of arcs in this paper:

Considering that some nodes are chosen as hubs and the rest are non-hubs, the connections between two non-hub nodes are considered in two ways: direct connections between nodes. Let Cij denote the unit routing cost from node i to node j and the route from node i to node j that travels on the path i-k-m-j where k and m are hub nodes, let Cik + Ckm + Cmj denote the unit routing cost in this case. See the possible forms in Figure 1.

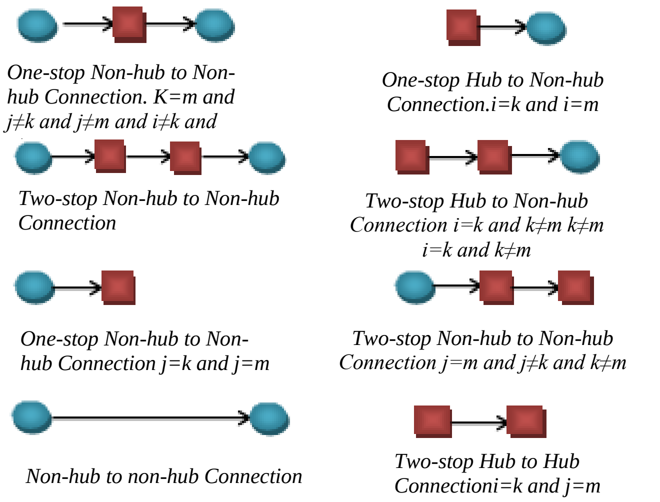

Figure 1: Different Structure of Hub Networks

1. arc between non-hub nodes that the cost of passing through the arc comes following:
   $C_{ij}= \beta * d_{ij} \quad \beta>1, \quad i,j \in$ non-hub
2. Arcs between non-hub and hub nodes that the cost of passing through the arc comes following:
   $C_{ik}=d_{ik} \quad i \in$ non-hubs, $k \in$ hubs
3. Arcs between two hubs. The cost of passing through this arc comes following:
   $C_{km}= \alpha * d_{km}, \alpha<1, k,m \in$ non-hub

As we confront with uncertainty in costs in productive and services environments, in this study with considering fuzzy circumstance, assuming uncertainty in costs seems logical. As showed in parameters introduction $\widetilde{f_k}$ ، $\widetilde{f_{hh}}$ ، $\widetilde{f_{hn}}$ و $\widetilde{f_{nn}}$ are set up costs that follow an asymmetric trapezium affiliation function. Demand in f^2 distance up to f^3 distance is uniform that has the most possibility for occurs. According to figure 3, in f^1 distance to f^2 distance is ascending and in f^3 distance to f^4 distance is descending.

For de-fuzzy we act like below. That usually considered σ 0.05 or 0.001.

$$\psi(\tilde{f}) = \frac{f^1+f^2+f^3+f^4}{4} - \sigma\left(\frac{f^3-f^2}{4}+\frac{f^4-f^1}{2}\right)$$

II-A. Assumptions

1. This model is a hub median problem.
2. This is a multiple allocation problem.in multiple allocation networks, a non-hub node can be allocated to more than one hub .
3. Each node can be connected to other node directly or go through the hub nodes.
4. The model is called endogenous that means the number of hub nodes is unclear and determined through a solving model.
5. For connecting two independent nodes, we should go through the two hub nodes.

The example of this model is shown in Figure 3.this figure shows that it can possible go from node 5 to node 2 through the 2-1-3-5 or 2-1-4-5 or 2-4-5 -routs. Rout selection is happened bye considering objective functions and costs. In this example, 1, 3, 4 are hubs and 1-3, 1-4, denote hub arcs.

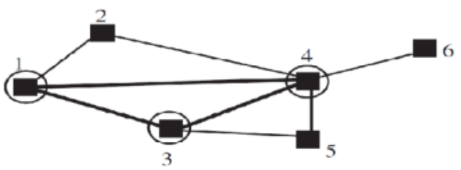

Figure 3: The Example of Model

In this model all kinds of cost,such as fixed costs of hubs, fixed costs of hub arcs, fixed costs of non-hub–hub connections, fixed cost of non-hub–non-hub connections are considered.

II-B. Definition of Indexes

i , *j*: set of non-hub hubs

k , *m*: set of hubs

III-B. Definition of Parameters

α : Bargain coefficient of relation between two hubs.

β: Penalty coefficient of relation between two non-hub hubs.

$\tilde{f}_k$: set up cost of a hub

$\tilde{f}_{hh}$: set up cost of an arc between two hubs

$\tilde{f}_{hn}$: set up cost of an arc between a hub and a non-hub points.

$\tilde{f}_{nn}$: set up cost a non-hub arc between two non-hub points.

w_{ij}: Flow that transfer between *i* and *j* hub.

d_{ij}: Distance between *i* and *j* hub.

IV-B. Definition of Variables

x_{ijkm} shows that there is a traffic between origin *i* to destination *j*. This variable takes the value 1. if we go through the hub k and m for getting from *i* to *j* otherwise it will have the value 0.

y_k is defined as a binary variable that takes the value 1 if the node k is a hub and otherwise it will have the value 0.

z_{km} If there is a hub arc between two hubs *m* , *k*, z_{km} takes the value 1 and otherwise 0.

z_{ik} if there is an arc between non-hub *i* and hub *k*, z_{ik} takes the value .

q_{ij} takes the value 1 if there is an arc between two non-hub nodes and otherwise it takes.

Model is as follows.

$$Min\ Z = \left(\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{m=1}^{n}(d_{ik}+\alpha d_{km}+d_{mj})w_{ij}x_{ijkm} + \sum_{i=1}^{n}\sum_{j=1}^{n}(\beta d_{ij}w_{ij})q_{ij}\right)$$
$$+\sum_{k=1}^{n} f_k y_k + \sum_{k=1}^{n}\sum_{m=1}^{n}\frac{1}{2} f_{hh} z_{km} + \sum_{i=1}^{n}\sum_{k=1}^{n} f_{hn} z_{ik} + \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} f_{nn} y_{ij}$$

Subject To:

$$y_{ij} + \sum_{k=1}^{n}\sum_{m=1}^{n} x_{ijkm} = 1 \qquad (1)$$

$$X_{ijkk} + \sum_{m\neq k}\left(X_{ijkm}+X_{ijmk}\right) \leq Y_k, \forall(i,j,k,m) \in x\ feas, i<j \qquad (2)$$

$$2z_{km} \leq y_k + y_m \qquad (3)$$

$$q_{ij} \leq y_{ij} + y_{ji} \qquad (4)$$

$$y_{ij} \in \{0,1\}$$

$$q_{ij} \in \{0,1\}$$

$$y_k \in \{0,1\} \quad \forall k \in V$$

$$x_{ijkm} \geq 0 \quad \forall(i,j,k,m) \in x\ feas, \quad i<j$$

$$w_{ij} \geq 0\ , \qquad i<j$$

The objective function (1) minimizes the costs for hub-hub connections, non-hub to hub connections, non-hub to hub connections through one hub, non-hub connections with passing through the two hubs nodes. We consider α coefficient as a discount factor in my cost. In this model the costs of direct connection between non-hub nodes are also considered. We also assume that there is a β (penalty coefficient cost) so that the direct route is selected when the cost of passing through the hub nodes is high. f_i is a fixed cost of considering node as a hub. For having a route between two hubs nodes that satisfies the system's needs, it requires a path with high capacity and resources that has been shown as f_{hh}. We have a cost for creation a route between a hub and a non-hub node that has been shown by

f_{hn}. The cost of direct arc between two non-hub nodes is shown as f_{nn}.

Constraints (1) assure that the route is unique whether it create directly or going through the hubs. Constraints (2) ensure that the flows between the hubs which have an arc between them are created. Creating hub arc for considering discount coefficient between *i* and *j*, when *i* and *j* be hubs, is ensured by Constraints (3).Constraints (4) describe that the traffic between non-hubs nodes take place when there is an arc between them.

III. PRESENT A NUMERICAL EXAMPLE

To illustrate our mathematical model thoroughly we present an example with using the CAB dataset that commonly used in hub location research. The fixed costs for the placement of the city as a hub, the cost of a route between two hub cities, the cost of a route between hub to non-hub cities, and the cost of a route between two non-hub cities are estimated. This problem is solved by CPLEX 10, Ci5 2.2GHz processor with 4GB RAM. Results of solving model with the different values of α and β are presented in Tables I and II.in this table α is a discount factor,β is a penalty coefficient and *P* denotes the number of cities that considered as a hub. *S* specifies the pairs of cities that are connected directly without passing through the hub. Cost shows a total cost of this model and CPU time indicate the solving time.

TABLE I: Summary of Test Results For α = 0.2

	β	P	S	Cost	CPU(s)
α = 0.2	1.2	2,4,6,12,13, 14,17,18 ,23	(7,8), (7,10), (7,11), (7,16), (8,11) ,(8,15), (8,19), (8,22), (10,16) ,(11,21), (19,22)	4486	16.532
	2	2,4,6,12,13 ,14,17,18, 23	(7,10), (10,16)	4506	16.407
	3	2,4,6,12,13 ,14,17,18, 23	(7,10)	4515	16.235

TABLE II: Summary of Test Results For α = 0.6

	β	P	S	Cost	CPU(s)
α = 0.6	1.2	2,4,13,14,19	(3,17),(3,18), (5,6),(6,9), (6,20),(7,8) ,(7,10), (7,11),(7,16), (8,11),(8,15), (8,23),(9,20), (10,16), (11,21)(12,22),(12,23), (15,23), (17,18), (22,23)	6852	16.509
	2	2,4,13,14,17,19	(6,9),(7,10), (9,20), (10,16) ,(12,22), (22,23)	6931	15.394
	3	4,13,14, 17,19, 23	(6,9),(7,10)	7092	15.601

The α values are 0.2 and 0.6 and β are 1.2, 2, and 3. For example, we solved the model with α =0.2 and β =1.2 in the result, the CPU time is 16.532s and the total cost is 4486 million unit cost. The cities that are considered as hubs :2,4,6,12,13,14,17,18,23 and the Pairs of cities that are directly connected: (19,22) , (11,21) , (10,16) , (8,22) , (8,19) , (8,15) , (8,11) , (7,16) , (7,11) , (7,10) , (7,8).

For example 7 and 8 are origin and destination nodes which are connected directly even though there is a penalty coefficient for direct connection. The direct route is selected because of higher transfer cost in passing through the hubs than direct connection. For going from node 7 to node 21, we should go through one or two hubs, in this example we go through the 12 and 18 hub nodes. In Table I and Table II it can be seen that with the same $\alpha$and increasing β, the total cost increases and direct connections between each pair of non-hub node decreases. It is reasonable, Because of increasing the penalty rates for direct connection to origin and destination, so crossing through the hubs spontaneously is selected. Necessarily increasing β does not lead to an increase the number of hubs. (in Table I Number of hubs is constant).In Table I and Table II it can be seen that with the same β and increasing α, the total cost increases and the number of hub cities decreases, because of passing through the hubs have a less discount value and this is reasonable. In this situation the cost of passing through the hub arcs increase and the number of hub nodes that have a direct link with the number of hub arcs decreases. Table I and Table II show that the number of direct arcs with increasing the value of α increases, because the cost of passing through the hub increases but the value of β is constant.

IV. CONCLUSION

In this paper, we study the hub median problem with multiple allocation of non-hub nodes to hub nodes, more over in this paper, it is possible to connect non-hub nodes directly. We solve this model for data set CAB25 and estimated costs. The results of our computational study lead us to the following conclusions: We first conclude that decreasing the penalty coefficient and constant discount factor caused to select more direct connections between non-hub nodes and with increasing the discount factor and same penalty coefficient, the more hub arcs are selected. This model is endogenous, that means the number of hubs are not clear and determined by solving the problem.

Hence, an interesting future research direction would be to derive stronger formulations for this problem. Other areas for future research include consideration another objectives such as maximizing service level and solving the multi objective model. It can be possible to consider direct connections between non-hubs in hub covering and hub center models.

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