

COMPARISON ACCURATE CALCULATE AND NUMERICAL SOLUTION OF BUCKLING THIN CYLINDRICAL SHAPE MEMORY ALLOY SHELL UNDER UNIFORM LOAD WITH D.Q.M

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ABSTRACT

The shape memory alloy has found too much significance among engineers and researchers due to their mechanical features behaviors and these alloys features are changed by temperature and tension. Also in correspondence to great mechanical strain made due to none linear and geometric features emerged in structure, encounter the equations solution with difficulty. In this research the numerical number has been presented based on the DQM basis to axisymmetric buckling of cylindrical shells made of shape memory alloys. First the solution incorporation presented then the numerical results are compared with exact solutions

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INTRODUCTION

The intelligent alloys materials mechanics has been under the specific attention during recent decades. As it is obvious from their names, these materials are from group of elements which have exclusive features. These characteristics are related o these elements phase martensite transformation. This feature allow them to have application in various range of industries as aerospace, measurement tools and medical implementations and etc. these materials have unusual figures which have the capability to recycle the great buckling forces in themselves and return to the primary shape, this characteristic and the form related to crystals shape in these materials toward temperature and tension. On the same basis we will be able to design materials

with high flexibility. Also these alloys will have low weight, compatible with environment, casting comfort and high force to weight ratio. To make a correct decision we have to analyze the behavior of these materials correctly and in following we will have a review about the features of these materials

The intelligent alloys have two phases, the temperature phase which is known the austenitic phase. And the martensite phase which is in two below form of DETWINNED and TWINED that you can see in figure number 1, during the cooling process in unloading type the materials will change from austenite twinned and the phase variation occurs in it and it passes its complete recycling shape path which is shown in figure 2. During this alteration no any deformation occurs.

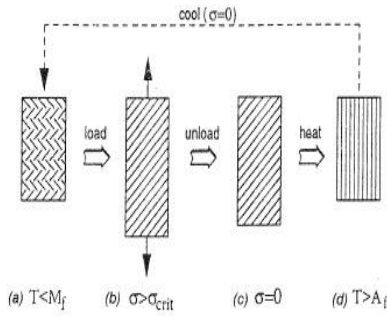


Figure 1; the phase alteration in A, B, C and martensite Detwinned and D in the austenite shape

During heating the part, it changes from the martensite to austenite shape. In the twinned martensite formation as we put force to the part, it will be able to be changed to Detwinned martensite. Now if the material put to the temperature A_f , the reverse phase alteration occurs in it and it follows its shape recycling complete path. This

process has been shown in figure 2. As the load put in the austenite phase, the material will start cooling, then the formed phase will be twinned and great buckling about 5 to 8% will occur in the material which will recycled by heating completely

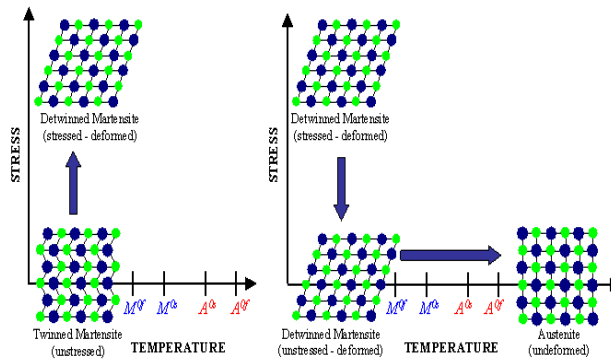


Figure2; the effect of intelligent alloys shape memory

Semi elastic feature

This feature occurs when we provide special mechanical loads to the part. The result of this loading is the Detwinned martensite complete phase. If the material put

is the high temperature of A_f , it will completely return to the primary shape from the made deformation. The piezo elastic effects are shown in figure 3 and the tension, strain graph also presented in the figure 4 as well

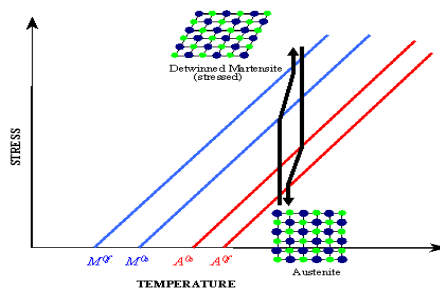


Figure3; the semi elastic behavior of intelligent alloys under loading

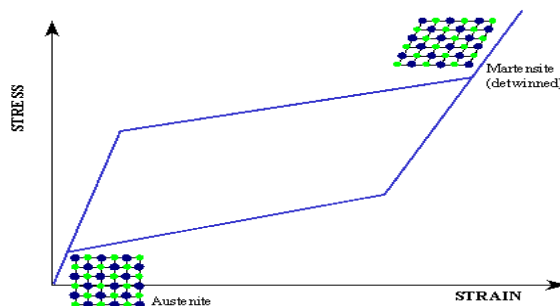


Figure4; the tension, strain semi elastic behavior in intelligent alloys

The super elastic feature

The term super elastic is used to define specific feature of intelligent alloys which find their primary shape if they are deformed or put under a load. The super elastic alloys could resist the strain 10 times more compare to usual spring without any deformation occurs in them. This unusual feature called super elastic, hence it is discussed as one of the features of intelligent materials 4

The energy lost feature

The hysteresis curve is describable by below parameters; The width of hysteresis graph $\Delta\sigma^1$ the tension domain (pressurizing, extension), the length of graph $\epsilon\Delta$ (the strain domain) is inside the substance elastic limit. The lost energy could be calculated in the close cycle of hysteresis by below equation as figure 5. In the graph when variables cut the motivation line in high condition, the substance is under the loading, it means that from austenite to martensite 5.

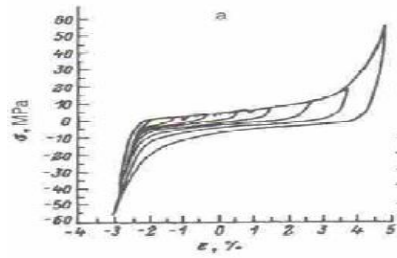


Figure 5; the strain- tension graph

The equations in intelligent alloys

The dominant law in intelligent alloys could be presented based on the deferential formation and by the basic alteration principals as below

$$DS = D(E, \xi, T)dE + \Omega(E, \xi, T)d\xi_s + \theta(E, \xi, T)dt$$

As the S is the tension second level (PIOLA KIRICHHOFF), the E is the strain factor and ξ is the internal variable which represent the phase transferring steps. D is the module for intelligent materials and Ω is the transference factor and θ is the thermal coefficient for intelligent alloys

The martensite ξ decimal function could be declared as below

$$2- \xi = \xi_s + \xi_T$$

ξ_T Shows the martensite decimal due to temperature and ξ_s is the representation of martensite decimal function based on the directed tension. The YANG module presented as the linear function from the martensite decimal function

$$3- D(E, \xi, T) = D(\xi) = D_a + \xi(D_m - D_a)$$

DM is the YANG module for intelligent alloys in martensite phase formation (Detwinned) and Da in YANG module is for representation of complete austenite shape. In the equation (1) with given function we can determine the relation between D and Ω by feature of materials with the remained maximum strain.

The remain maximum strain ϵ_1 calculated by the experimental equation and by alteration of substance to martensite shape of Detwinned ($\xi_s = 1$) and then we unload the substance in the lower temperature of austenite beginning temperature, hence as BRAUNSON presented, the transference tensor is determined as the function of YAUNG module as below

$$4- \Omega(\xi) = -\epsilon_1 D(\xi)$$

By consideration of parameters as linear based on ξ . The law of composite materials equation will be as below

$$\sigma - \sigma_0 = D(\xi)\epsilon - D(\xi_0)\epsilon_0 + \Omega(\xi)\xi_s - \Omega(\xi_0)\xi_{s0} + \theta(T - T_0)$$

As $(T_0, S_0, \epsilon_0, \xi_0, \xi_{s0})$, is representation of the substance primary shape 4

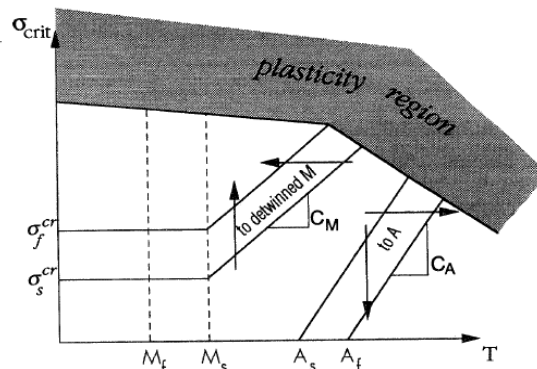


Figure 6; the phase 2 transformation kinetics graph

For the martensite transformation formation and $T > M_s$

$$6- \sigma_s^{cr} + C_m(T - M_s) < S < \sigma_f^{cr} + C_m(T - M_s)$$

$$7- \xi_s = \frac{1 - \xi_{s0}}{2} \cos\left\{ \frac{\pi}{\sigma_s^{cr} - \sigma_f^{cr}} (s - \sigma_f^{cr} - C_m(T - M_s)) \right\}$$

$$+ \frac{1 + \xi_{s0}}{2}$$

For the transformation to martensite

$$T < M_s \text{ And } \sigma_s^{cr} < S < \sigma_f^{cr}$$

$$8- \xi_s = \frac{1 - \xi_{s0}}{2} \cos\left\{ \frac{\pi}{\sigma_s^{cr} - \sigma_f^{cr}} (s - \sigma_f^{cr}) \right\} + \frac{1 + \xi_{s0}}{2}$$

$$9- \xi_T = \xi_{T0} - \frac{\xi_{T0}}{1 - \xi_{s0}} (\xi_s - \xi_{s0}) + \Delta T_\xi$$

If $M_f < T < M_s$

$$\Delta T_\xi = \frac{1 - \xi_{T0}}{2} \cos[a_m(T - M_f) + 1]$$

Other wise 11- $\Delta T_\xi = 0$

For the transformation condition to austenite $T > A_s$ and

$$C_a(T - A_f) < S < C_a(T - A_s)$$

$$\xi = \frac{\xi_0}{2} \{ \cos[a_A (T - A_s - \frac{s}{C_A}) + 1] \}$$

$$\xi_s = \xi_{s0} - \frac{\xi_{s0}}{\xi_0} (\xi_0 - \xi) \quad 12- \xi_T = \xi_{T0} - \frac{\xi_{T0}}{\xi_0} (\xi_0 - \xi)$$

The a_A, a_m parameters are explained as follow; 13-

$$a_m = \frac{\pi}{M_s - M_f}, \quad a_A = \frac{\pi}{A_f - A_s}$$

The C_M, C_A parameters explain the relation between the temperature and critical tension for composite materials phase transference based on the BRAINSON model. As it is described in the figure in higher temperature of MS, to transfer to martensite phase and for lower temperature of mentioned degrees, the tension remain stable 6

The general form dominant in the equation based on the DONNEL equation would be as below

$$14- D_{11} \frac{d^4 w}{dx^4} + N \frac{d^2 w}{dx^2} + \frac{A_{22}}{R^2} (1 - g^2) = 0$$

The accurate solving method for dominant equation in cylindrical shells buckling

The general solution for mentioned equation could be found based on the expositor and root determination equations

$$(s^4 + \frac{N}{D_{11}} s^2 + \frac{A_{22}}{D_{11} R^2} (1 - g^2)) = 0,$$

The roots for this equation is as below

15-

$$S_{1,2,3,4} = \pm i \left[-\frac{N}{D_{11}} \pm \left[\left(\frac{N}{2D_{11}} \right)^2 - \frac{A_{22}}{D_{11} R^2} (1 - g^2) \right]^{1/2} \right]^{1/2}$$

The result of equation will be as below in general form

$$16- W = c_1 \sin(S_1 x) + c_2 \cos(S_1 x) + c_3 \sin(S_3 x) + c_4 \cos(S_3 x)$$

By regarding boundary condition in simple, we will have in two sides as below

$$W(x=0) = 0, \quad \frac{d^2 w}{dx^2}(x=0) = 0, \quad W(x=L) = 0, \quad \frac{d^2 w}{dx^2}(x=L) = 0$$

$$\begin{cases} \left[\frac{N}{2D_{11}} + \left[\left(\frac{N}{2D_{11}} \right)^2 - \frac{A_{22}}{D_{11} R^2} (1 - g^2) \right]^{1/2} \right]^{1/2} = \frac{n\pi}{L}, \\ \left[\frac{N}{2D_{11}} - \left[\left(\frac{N}{2D_{11}} \right)^2 - \frac{A_{22}}{D_{11} R^2} (1 - g^2) \right]^{1/2} \right]^{1/2} = \frac{n\pi}{L}, \end{cases}$$

By fulfillment of boundary condition in the formation condition, we will have 2 side fixtures as below

$$W(x=0) = 0, \quad \frac{dw}{dx}(x=0) = 0, \quad W(x=L) = 0, \quad \frac{dw}{dx}(x=L) = 0$$

The buckling equation total response would be as below

$$W = -\frac{S_3}{S_1} \sin(S_1 x) + \left(\frac{\sin(S_3) - \frac{s_3}{s_1} \sin(S_1)}{\cos(S_3) - \cos(S_1)} \right) \cos(S_1 x) + \sin(S_3 x) - \left(\frac{\sin(S_3) - \frac{S_3}{S_1} \sin(S_1)}{\cos(S_3) - \cos(S_1)} \right) \cos(S_3 x),$$

If the support formation condition is simple at fixture we will have

$$W(x=0) = 0, \quad \frac{d^2 w}{dx^2}(x=0) = 0, \quad W(x=L) = 0, \quad \frac{dw}{dx}(x=L) = 0$$

The behavior response for buckling equation will be calculated as below

$$W = (\sin(S_3 X) - (\sin(S_3 L) / \sin(S_1 L)) \sin(S_1 X)),$$

The numerical solving method

The dominant equations by DQM method in all points of separated grates and by fulfillment of boundary condition appropriate to programming, the calculation will be done By performing below procedures, the dominant equation would be as below

$$D_{11} [C^4 W_{ij} + N [C^2 W_{ij} + \frac{A_{22}}{R^2} (1 - g^2) W_{ij}] = 0,$$

$$D_{11} [c^4] + \frac{A_{22}}{R^2} (1 - g^2) [C^2]_{inv} = -N [C^2],$$

$$18- [C^2]_{inv} [D_{11} [c^4] + \frac{A_{22}}{R^2} (1 - g^2)] = N,$$

The N value is the critical load in buckling

The boundary condition in the form of DQM method is as below

Two side of substance in a simple

$$W(x=0) = 0, [1 \ 0 \ 0 \ \dots \ 0] W_{ij} = 0, \quad M_x(x=0) = 0,$$

$$[C_{2j}^2] W_{ij} = 0, \quad W(x=L) = 0, [1 \ 0 \ 0 \ \dots \ 0] \times W_{ij} = 0, \quad M_x(x=L) = 0,$$

$$19 \quad M_x(x=L) = 0, [C_{n-1,j}^2] W_{ij} = 0, -$$

$$M_x(x=L) = 0, [C_{n-1,j}^2] W_{ij} = 0,$$

Two side of a fixed support

$$W(x=0) = 0, [1 \ 0 \ 0 \ \dots \ 0] W_{ij} = 0, \quad \frac{dw}{dx}(x=0) = 0,$$

$$[C_{2j}^2] W_{ij} = 0, \quad W(x=L) = 0, [1 \ 0 \ 0 \ \dots \ 0] W_{ij} = 0,$$

20-two side of a simple fixed

$$W(x=0) = 0, [1 \ 0 \ 0 \ \dots \ 0] W_{ij} = 0, \quad \frac{dw}{dx}(x=0) = 0,$$

$$[C_{2j}^2] W_{ij} = 0, \quad W(x=L) = 0, [0 \ 0 \ 0 \ \dots \ 1] W_{ij} = 0,$$

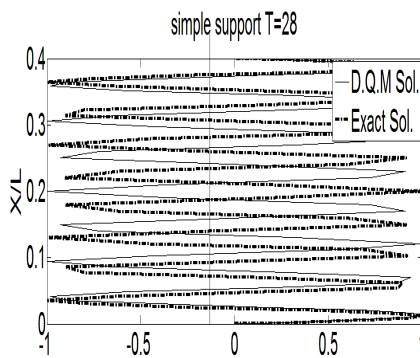
$$(21) \quad M_x(x=L) = 0, [C_{n-1,j}^2] W_{ij} = 0,$$

Discussion and results evaluation

By comparing the results obtained through the accurate calculation and numerical solving of dominant equation in buckling at cylindrical shells we come to this result that the convergence is with very low error percentage divergent and also by comparing the buckling graph obtained through calculation we see that results have perfect conformity. Some of the provided results are as below

Table1; evaluation of critical

buckling load convergence in thin layer cylinders made of intelligent alloys at 28 centigrade for simple support



Simple support in temperature 28C				
PRIMARY LOAD (kpa)	STEP	Load through analytic method(kpa)	Final put load (kpa)	Critical load through method (kpa)DQM
300	100	486	500	494.3
400	50	486.08	500	
450	5	489.09	495	
480	1	490.9	492	
487.1	0.1	491.32	491.3	
490	0.01	491.33	491.29	

Figure 8; the thin layer cylinder buckling moods graph in the simple support condition obtained through the accurate calculation and DQM method in temperature 28 C

TABLE2; evaluation of convergence in critical buckling load put in to thin layer cylinder made of intelligent alloys at 40 centigrade for simple support

Simple support in temperature 40C				
PRIMARY LOAD (kpa)	STEP	Load through analytic method(kpa)	Final put load (kpa)	Critical load through method (kpa)DQM
300	100	388.34	400	375.8
350	5	378.87	380	
370	1	377.94	378	
377	0.1	377.9	377.9	

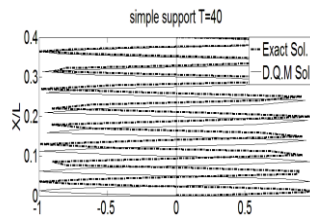


figure9; the thin layer cylinder buckling graph in simple support condition obtained through accurate calculation and DQM method in 40C

TABLE3; The comparison of obtained buckling load values trough DQM method by accurate calculation in different material formation in austenite simple support condition

Critical Load table simple(kpa)			
Mode no.	17	18	19
Exact Solution	607.85	617.75	634.56
Present D.Q.M	607.85	618.26	631.75

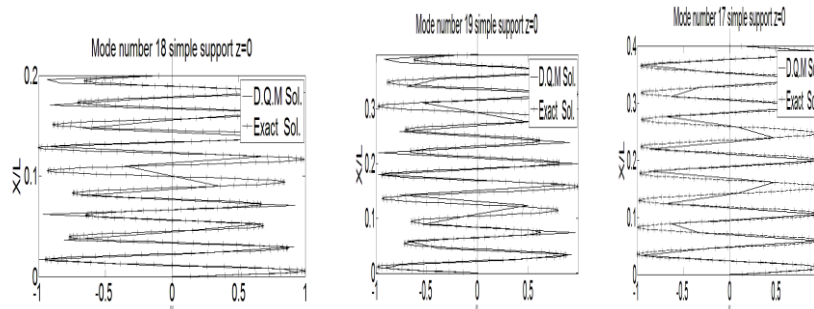


Figure10; the thin layer cylinder buckling moods graph in simple support condition for austenite obtained through accurate calculation and DQM method

Table 4; obtained buckling load value comparison through DQM method by accurate solving in different moods of martensite simple support

Critical Load table simple(kpa)			
Mode no.	17	18	19
Exact Solution	263.4	267.69	273.76
Present D.Q.M	263.4	267.91	273.76

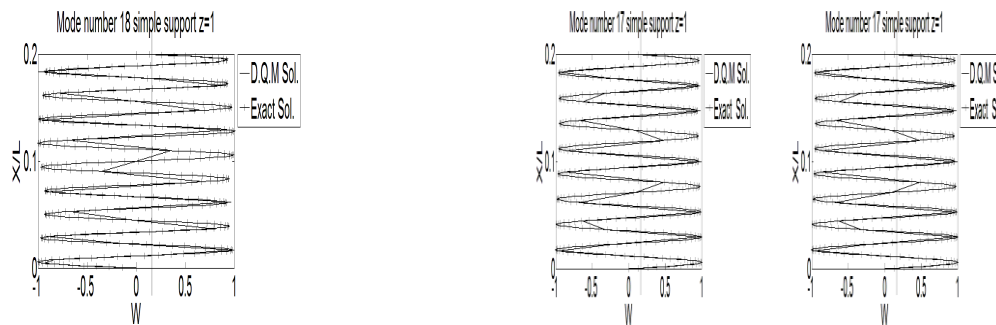


Figure11; the thin layer cylinder buckling mood graphs in austenite simple support obtained through accurate calculation and DQM method

Table5; obtained buckling load values comparison through DQM method by accurate solving in different moods in martensite fixed support

Critical Load table			
Mode no.	15	16	20
Exact Solution	264.5	361.5	458.5
Present D.Q.M	264.49	361.96	458.76

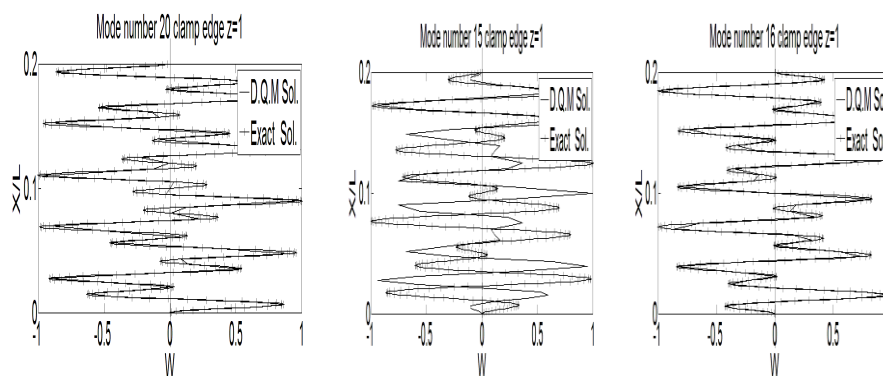


Figure12; the thin layer cylinder buckling moods graph in austenite fixed support condition obtained through accurate calculation and DQM method

CONCLUSION

In the performed research, first we discussed about the memory shape alloys and then we talked about their applications, then the phase alteration kinetics relations and tension- strain behavior presented by LIANG ROGERS and BRINSON. By the use of behavioral models presented and DONNEL theory and extracting balance equations we evaluated the buckling behavior. It should be noted that we avoided the effect of sectorial forces in this issue and also it is considered that loading is done symmetrical. In the continuity we present the accurate solving method and also the equations solution convergence and by comparing them with numerical DQM solution, the assessment fulfilled about accuracy. Then comparison made with other sources to evaluate authenticity

The signs reference

The signs reference brought before the references, first the English signs alphabetically and then the Greece signs as well

A; space square meter

H; thickness

Greece signs

ξ ; martensite; decimal function

S; martensite decimal function due to tension effect

T; martensite decimal function due to temperature effect

REFERENCE

1-Ashiqur Rahman1. M , M.A.K Chowdhuri2, Finitedifferenceanalysisof short smacolumnswithtension-

compressionasymmetry International Conference on Mechanical Engineering 2007

2- Sia Nemat-Nasser, Jeom Yong Choi, Jon B. Isaacs, and David W. Lischer, Experimental Observation of High-rate Buckling of Thin Cylindrical Shape-memory Shells, University of California, San Diego, Center of Excellence for Advanced Materials, Edited by William D. Armstrong, Proceedings of Spie Vol. 5761

3- W Fang and J A Wickert, Post buckling of micromachined beams, Department of Mechanical Engineering, Camegie Mellon University, August 1994

4- L.C BRINSON ,R.LAMMERING,Finite Element Analysis Of The Behavior Of Shape Memory alloys and Their Applications,North western university 17 may 1993

5- L.C BRINSON, One – Dimensional Constitutive Behavior Of Shape Memory Alloys : Thermomechanical Derivation With Non-Constant Material Function And Redefined Martensite Internal Variable,Journal of intelligent material and structure vol 4 april 1993

6- M.A. Rahman, Postbuckling characteristic of the short superelastic shapememory alloy columns- experimental and quantitatives analysis, Department of Mechanical Engineering, Bangladesh University of Engineering & Technology, Int. J. of Applied Mechanics and Engineering, 2006, vol.11, No.4, pp.941-955

8-Loubna Hessissen, Mohamed Essaaidi, Thermomécanical Behavior of Polycrystalline Shape Memory Alloys in Martensitic Transformation,European Journal of Scientific Research ISSN 1450-216X Vol.23 No.3 (2008), pp.474-481

9- Definition of a Shape Memory Alloy , S.M.A Group
10- Stoeckel, Yu, Super elastic Ni Ti Wire Stoeckel, Yu
Wire Journal International March 1991,Wire journal
International, March 1991, pp. 45-50
11- Smart material Exploring shape memory alloy
Georgia institute 2007

12- Tautzenberger, Stoeckel,The Use of Shape Memory
Alloys in Switchgear Technology,1984
13- C. M. WANG*, Z. Y. TAY†,§ and A. N. R.
Chowdhuary‡, Examination of cylindrical shell theories
for buckling of carbon nanotubes, International Journal
of Structural Stability and Dynamics, Vol. 11, No. 6
(2011) 1035_1058