

Optimal allocation of distributed generation using an analytical method with consideration of technical and economic parameters

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ABSTRACT

In order to benefit maximally from positive features of Distributed Generation (DG) supplies, the finding of location (siting), and determination of optimal capacity of these supplies are inevitable in power distribution networks so that the researches indicate that if this important process is not considered in installation of DG supplies, not only the given positive goals and features are not fulfilled in installation of DG supplies, but also it is probably led to the worse status in installation of power supplies than the past as well. As a result, determination of optimal size and position of power distributed generation supplies is assumed as an important point.

An analytical technique has been proposed in this essay for siting and sizing of DG supplies. Technical and economic aspects have been simultaneously considered in the aforesaid method and it is aimed at maximization of profit due to installation of these supplies. Initially, all parameters have been expressed as a function of DG power and then the optimal siting and sizing of DG is determined by employing the suggested method. The studied supplies are of wind and solar types. Simulations are implemented in MATLAB software and the studied systems include IEEE standard 33 and 69- bus networks.

Original Article:

Received 23 Dec. 2014

Accepted 30 Dec. 2014

Published 30 Mar. 2015

Keywords:

Distributed Generation Supply,
Losses,
Economic Parameters, Technical
Parameters, Voltage Profile,
Siting

1- Introduction

The studies indicate that 13% of losses in power supplies are related to distribution system. Losses in this system not only cause wasting of electrical energy, but also lead to occupying the capacity of transformers and transfer lines. Double losses at the time of peak charge consumption (demand) increase the need to investment in development of power plants and transfer network [1]. Of methods employed for reducing losses one can refer to using DG supplies. In addition, the economic factors should be also taken into consideration at time of installation of these supplies. Therefore, with appropriate utilization from DG units (siting and suitable sizing for them) one can benefit from economic advantages of their presence in the network in addition to technical advantages. Siting and optimal sizing for DG supplies are divided into four categories

based on the type of the given parameter: Siting and sizing based on economic parameters, siting and sizing based on professional and technical factors in network, siting and sizing based on determination of optimal capacity as a composition of economic and technical parameters, and the fourth class is the siting and sizing for maximum permittivity of these elements in the network. Those studies, which have dealt with siting and optimum sizing for DG supplies, only notice the profit and cost caused by them. Technical factors and effect of installation of these supplies are not considered in these studies. Thus, technical advantages due to installation of these supplies could not be achieved by means of these studies and even it is possible to technical factors to be undermined. Some examples of these studies may be observed in references [2, 3, 4]. Only technical factors are noticed in the second class. The

technical factor that is seen in most of studies on siting and sizing is factor of losses. The examples of this group of studies can be seen in [5, 6, and 7]. Nonetheless, in some studies like [8, 9], other factors including reduced level of short-circuits, improving voltage profile, harmonic reduction, rising reliability have been considered as target function. The foremost challenge in these studies is lack of cost-effectiveness governing over their results. Third category explores the siting and sizing for DG supplies based on technical and economic factors at the same time. This class of studies is composed of the abovementioned methods. In this group of studies technical and economic advantages are considered [10,11].

The forth class determines maximum penetration of DG supplies in the network and DG penetration as ratio of injected energy of DG to the network. The rate of this penetration varies in any country and governments determine it [12]. The process of siting and optimal sizing of DG supplies in some studies e.g. [13,14] were mentioned as increase in penetration of these supplies in the network and approaching to maximum value determined by the governments. Overall, these studied are not intended to improve technical parameters of network by installation of supplies, but these studies are aimed to generate power in upstream networks within the framework of the existing constraints.

An optimal technique should be employed for siting and sizing of DG supplies. Various methods are adapted in several ways. Generally, these methods can be divided into three classes. First group includes analytical techniques like 2/3 feeder technique [15], [7,16] and sensitivity analysis. In sensitivity analysis method, the sensitivity of losses in real power is used to ratio of reactive power (for capacitor) and active power (for DG supply) as indices to determine optimal siting of them. The bus with higher calculated index will possess the best and optimal siting for installation of supplies [17]. The analytical technique has been used for siting and sizing of DG supply in references [18], [19], and [20]. Losses are the target function in these essays but it differs from them in that in studies of [18] and [20] siting has been done for various types of DG while in reference [19] this analysis has been done for a type of DG unit that is capable to inject active and reactive powers. Similarly, in references of [18] and [19] the siting has been done only for a DG supply while in [20] it has been implemented for more than one DG units. The second group comprises of meta-heuristic methods such as Genetic Algorithm (GA), Particle Swarm Optimization (PSO), Ant Colony Optimization technique etc. Whereas, reducing losses is considered as one of paramount reasons for DG installation. In [21] siting and sizing of DG have been implemented to reduce losses by means of CFPSO algorithm. Reference [22] has given the composition of PSO and GA together for optimal siting of DG supply in radial system. In this essay, Genetic Algorithm determines the appropriate siting for DG installation and also PSO algorithm identifies DG optimal

capacity. In this essay, target function includes losses and voltage profile. Third class of heuristic method is to explore all possible states. Heuristic methods are intuitive and consist of ease of perception and ease of implementation compared to analytic and classic techniques, but they need consuming long time in order to achieve favorable output. As an example of this technique, reference [23] has utilized this technique for siting simultaneously for several DG supplies.

In the present essay, siting and sizing in DG supplies are aimed at improving technical and economic parameters simultaneously. The given technical parameters include losses and voltage profile. Likewise, the studied economic factors including sale of energy, the profit due to reduced losses, cost of primary investment, equipment repairs and maintenance cost, substitution and replacement costs of equipments, and fuel cost. All of costs are written according to DG power and achieved by means of mathematical rules so the siting and optimal sizing DG.

2- Formulation of problem

As it already mentioned, in this essay we intend to maximize income acquired by installation of DG supplies and target function includes the profit derived from sale of energy to upstream network, the profit as a result of reducing losses, cost of primary investment, substitution cost, cost of repairs and maintenance of equipments and cost of fuel for DG supply. Thus, target function may be assumed as difference of sum of profits from sum of costs. Each of these functions is separately examined in the following and finally target function will be introduced.

2-1- Profit functions

- The profit as result of sale of energy

Due to sale of produced energy from DG supply to the network and lack of buying energy from the created upstream network, this profit is derived from the following formula:

$$C_R^{DG} = \sum_{t=1}^{8760} k \times E_{DG} \quad (1)$$

In this formula, C_R^{DG} denotes the profit acquired by sale of energy for a year, k is equivalent price of energy per hour t (\$/MWh), and E_{DG} as sold energy by DG (MWh).

- The profit acquired by reducing losses

As it implied, reduced losses in the system is one of the advantages of installation of DG supplies. Reduction of losses is equal to less wastage of energy and lesser purchase of energy. The profit acquired by reduction of losses is defined as follows:

$$C_{Loss} = (E_{Loss_before} - E_{Loss_after}) \times k \quad (2)$$

Where in this formula, k denotes energy in t hour, and E_{Loss_before} and E_{Loss_after} are losses (MW) before and after installation of DG supply, respectively. Also active losses of the system are acquired according to the following formula as well:

$$P_L = \sum_{i=1}^N \sum_{j=1}^N [a_{ij}(P_i P_j + Q_i Q_j) + \beta_{ij}(Q_i P_j - P_i Q_j)] \quad (3)$$

In this formula:

$$a_{ij} = \frac{R_{ij}}{|V_i||V_j|} \cos(\delta_i - \delta_j) \quad (4)$$

$$a_{ij} = \frac{R_{ij}}{|V_i||V_j|} \sin(\delta_i - \delta_j) \quad (5)$$

Where, P_j and P_i are i^{th} and j^{th} active bus powers, Q_j and Q_i as i^{th} and j^{th} reactive bus powers respectively, and N is number of buses in system.

$$P_i = P_{G_i} - P_{D_i} \quad (6)$$

$$Q_i = Q_{G_i} - Q_{D_i} \quad (7)$$

P_{G_i} denotes produced active power in i^{th} bus and P_{D_i} as consumed power of i^{th} bus. As a result, Q_{G_i} represents produced reactive power and Q_{D_i} , is consumed reactive power in i^{th} bus.

r_{ij} element (i,j) is the real part of impedance matrix. V_i and V_j are values of voltage in i^{th} and j^{th} buses and δ_i a δ_j are angles of voltage in i^{th} and j^{th} buses.

2-2- cost functions

• Annual investment cost

Capital Return Factor (CRF) coefficient is used for calculation of annual rate of primary investment cost that depends on interest rate and project lifetime.

$$CRF = \frac{r(1+r)^{ny}}{(1+r)^{ny} - 1} \quad (8)$$

In this formula, r denotes interest rate, and n_y is equal year of capital return (recovery).

Annual cost of primary investment DG is defined as follows:

$$C_{DG}^{cap} = CRF \times C_{DG}^c \times P_{DG} \quad (9)$$

In this formula, CRF denotes the capital return factor, C_{DG}^c is cost of installation of DG supply per \$/MW and P_{DG} is capacity of DG based on MW.

• Substitution cost

The lifetime of a project may be longer than useful life of the installed supply and for this reason the substitution cost is proposed for annual use and it is calculated according to following formula:

$$C_{DG}^{rep} = C_{DG}^{rep} \times P_{DG} \times \gamma_{DG} \times \lambda_{DG}(r, n_{DG}) - \xi_{DG} \lambda_{proj}(r, n_{proj}) \quad (10)$$

In this formula, C_{DG}^{rep} denotes substitution cost for DG supply \$/MW, P_{DG} is capacity of installed DG in MW, γ_{DG} is substitution coefficient of installed DG supply, λ_{DG} as floating useful life factor of DG, ξ_{DG} is extra value of installed DG, λ_{proj} as survival floating factor (SFF) of project.

• Repairs and maintenance cost

This cost is also derived annually and according to the following formula:

$$C_{DG}^{O\&M} = C_{DG}^{O\&M} \times P_{DG} \times T_{DG} \quad (11)$$

In this formula, T_{DG} denotes DG performance coefficient and it expressed in hours, $C_{DG}^{O\&M}$ is also the cost relating to repairs of equipments in \$/MWh.

• Fuel cost

DG supplies include wind, solar, and diesel supplies etc. No cost will be paid for fuel in wind and solar types of resources, but in diesel type of supply, some cost is paid for their consumed fuel and this cost is calculated as follows:

$$C_{DG}^f = \sum_{D=1}^{365} \sum_{t=1}^{24} E_{DG} \times K_2 \times C_{DG}^f \quad (12)$$

$$K_2 = \frac{1 + \frac{\theta}{\rho}}{\eta \times \gamma}$$

(13)

θ is ratio of DG energy conversion, ρ is ratio of energy to produced heat, η is DG efficiency, and γ is net thermal value of the fuel consumed by DG.

2-3- Target function

The target function is equal to the income produced due to DG installation that is defined as follows:

$$OF = \text{Sum of profit functions} - \text{Sum of cost functions} \quad (14)$$

$$OF = C_R^{DG} + C_{Loss} - C_{DG}^{cap} - C_{DG}^{rep} - C_{DG}^{O\&M} - C_{DG}^f \quad (15)$$

Installation of DG supply and siting and sizing of it are aimed at achieving further profit; therefore, the maximum value of target function should be determined. Utilization of derivative in mathematics is one of the techniques for determining extremum of a function so that we may calculate target function in critical points in order to determine its maximum value. Thus, if Eq. (15) can be written as an exponential function with power of the installed DG in i^{th} bus, the extremum of the function may be determined by means of mathematical rules.

3- Suggested technique for siting and sizing of DG capacity

It has been implied in reference [24] that if we can install DG on all of buses in such a way that the produced active and reactive power of DG is the same as the consumed active and reactive power in the same bus; namely, the consumed power for any bus to be produced in the same consumption site since the current is not transferred in these lines thus total losses of the system will be equal to zero and this the best mode that occurs for the system. In this mode, DG power coefficient is equal to charge power coefficient.

$$a_i = \frac{P_i}{\sqrt{P_i^2 + Q_i^2}} \quad (16)$$

But, in practice installation of DG on all buses is not feasible due to economic issues and this fact that all buses could not be installed. For this reason, we suppose that power coefficient of DG supply is equal to total power coefficient in the network. In this case, DG can compensate for total charge power of the network [19].

$$a_i = \frac{P_T}{\sqrt{P_T^2 + Q_T^2}} \quad (17)$$

P_T and Q_T are sum of active and reactive powers for consumed charges of network. In this essay, this technique has been adapted to calculate DG supply power coefficient. The power coefficient close to optimal is calculated by using this method.

In reference [14], an analytical technique has been expressed for siting and sizing of a DG supply in order to improve parameter of losses. It has been assumed in this essay that a DG supply with P_{DG} capacity has been installed in i^{th} bus. Then, the injected power by i^{th} bus varies based on Exp. (18) and (19):

$$P_i = P_{DG_i} - P_{D_i} \quad (18)$$

$$Q_i = Q_{DG_i} - Q_{D_i} = \alpha P_{DG_i} - Q_{D_i} \quad (19)$$

Moreover, size and angle of voltage of buses also vary after installation of DG supply and as a result losses of coefficients α_{ij} and β_{ij} are changed in this equation, but due to small amount of these changes we can ignore them.

Thus, the equation of losses is changed as Exp. (20):

$$P_L = \alpha_{ii}(P_i^2 + Q_i^2) + \sum_{i=1}^N \sum_{j=1, j \neq i}^N \left[\alpha_{ij}(P_i P_j + Q_i Q_j) + \beta_{ij}(Q_i P_j - P_i Q_j) \right] \quad (20)$$

As we know, we should explore the points in which tangent line on the given diagram is horizontal in order to find extremum of a single variable function. Similarly, we explore function values for the boundary point of their range as well as the points in which there is no first-order derivative. In this essay, Eq. (20) is differentiated in respect of P_{DG_i} i.e. unknown parameter in function of losses and its product is assumed as zero and therefore DG capacity is acquired. The calculated figure is equal to optimal capacity of the installed DG in i^{th} bus.

The analytical method that has been used in this essay based on this fact but differs from it in that the target function of the present essay includes economic parameters and quantization is done in order to maximize the profit. Therefore, if we suppose DG is present in i^{th} then EQs. (18-20) satisfy. Thus, the target function is rewritten as follows:

$$OF = \left\{ \sum_{t=1}^{8760} k \times E_{DG,i} \right\} + \left\{ (E_{Loss_before} - E_{Loss_after}) \times k \right\} - \{ CRF \times C^c_{DG} \times P_{DG,i} \} - \{ C^{rep}_{DG} \times P_{DG,i} \times \gamma_{DG} \times \lambda_{DG}(r, n_{DG}) - \xi_{DG} \lambda_{proj}(r, n_{proj}) \} - \{ C^{O\&M} \times P_{DG,i} \times T_{DG} \} - \left\{ \sum_{D=1}^{365} \sum_{t=1}^{24} E_{DG,i} \times K_2 \times C^f_{DG} \right\} \quad (21)$$

Also, P_{Loss_after} varies as follows:

$$P_{Loss_after} = \alpha_{ii} [(P_{DG_i} - P_{D_i})^2 + (\alpha P_{DG_i} - Q_{D_i})^2] + \sum_{i=1}^N \sum_{j=1, j \neq i}^N \left[\alpha_{ij} ((P_{DG_i} - P_{D_i}) P_j + (\alpha P_{DG_i} - Q_{D_i}) Q_j) + \beta_{ij} ((\alpha P_{DG_i} - Q_{D_i}) P_j - (P_{DG_i} - P_{D_i}) Q_j) \right] \quad (22)$$

As it implied, if we differentiate the target function in respective of Injected power of installed DG on i^{th} bus and suppose the product equal to zero then the maximum value (extremum) of target function will result. It has been assumed in this essay that in order to simplify these equations that The installed DG supply is of wind or solar types so no cost is spent for its fuel and additionally we suppose the useful life of project as identical with DG supply and the cost of substitution has been ignored. Thus we will have:

$$\frac{\partial OF}{\partial P_{DG_i}} = 0 \quad (23)$$

$$\frac{\partial OF}{\partial P_{DG_i}} = \sum_{t=1}^{8760} K \times 8760 - k \times 8760 \times \frac{\partial P_{Loss_after}}{\partial P_{DG_i}} - \{ CRF \times C^c_{DG} \} - \{ C^{O\&M} \times T_{DG} \} = 0 \quad (24)$$

If we substitute $\frac{\partial P_{Loss_after}}{\partial P_{DG_i}}$ in this formula, Eq. (24)

varies as follows:

$$\frac{\partial OF}{\partial P_{DG_i}} = \sum_{t=1}^{8760} K \times 8760 - k \times 8760 \times \{ 2(1 + \alpha^2) \alpha_{ii} P_{DG,i} - 2 \sum_{j=1}^N \alpha_{ij} (P_j + \alpha Q_j) + \beta_{ij} (\alpha P_j - Q_j) \} - \{ CRF \times C^c_{DG} \} - \{ C^{O\&M} \times T_{DG} \} = 0 \quad (25)$$

Therefore, the capacity of installed DG in i^{th} bus is equal to:

$$P_{DG_i} = \frac{2 \times k \times 8760 \sum_{j=1}^N \{ \alpha_{ij} (P_j + \alpha Q_j) + \beta_{ij} (\alpha P_j - Q_j) \}}{2 \times k \times 8760 (1 + \alpha^2) \alpha_{ii}}$$

$$\frac{+\sum_{t=1}^{8760} K \times 8760 - \{CRF \times C^c_{DG}\} - \{C^{O\&M} \times T_{DG}\}}{2 \times k \times 8760(1 + \alpha^2)\alpha_{ii}} \quad (26)$$

The optimal capacity of installed DG in any bus is determined by Eq. (26). As a result, the optimal capacity of DG is computed for all of buses. Therefore, quantity of the calculated capacities is the same as number of buses. Hence, the speed of calculation of response is high in this technique. At the next phase, we obtain the profit due to optimal installation of DG in any bus by means of optimal capacity derived for each of buses and that bus is determined as optimal bus that has the maximum profitability and its equivalent capacity is assumed as optimal capacity. The flowchart of the given algorithm is shown in Figure 1.

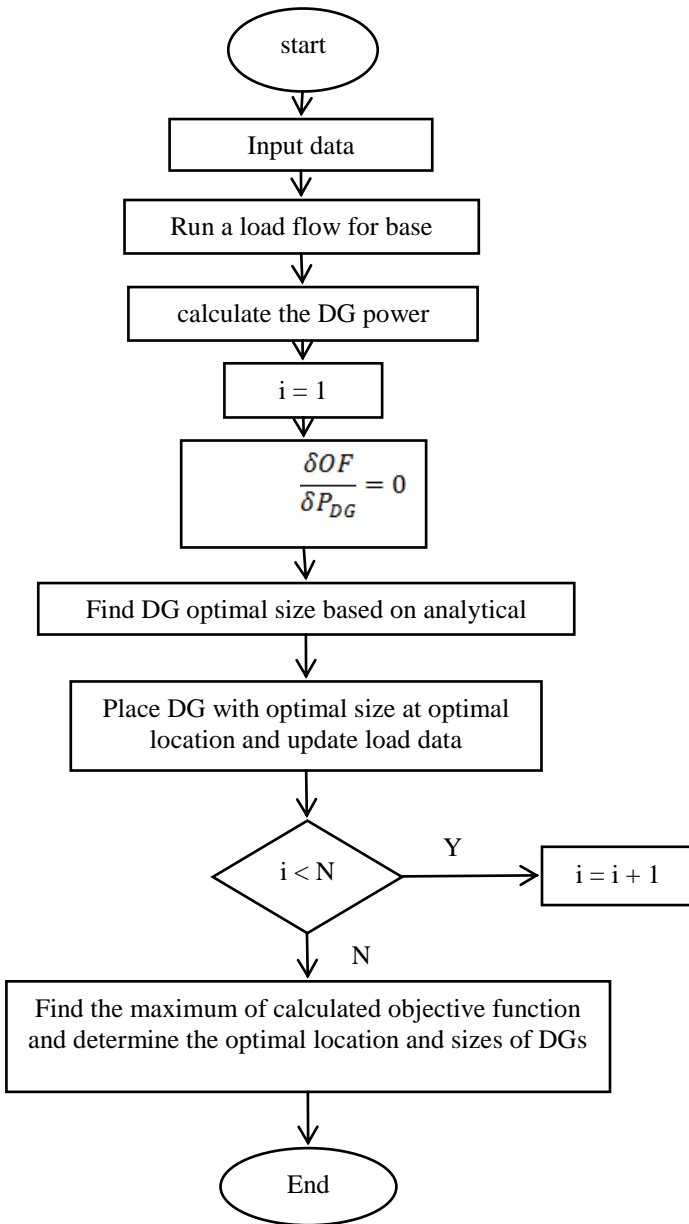


Figure 1. Flowchart of suggested technique

4- Introducing specifications of the studied systems

The studied systems are IEEE standard 33- and 69- bus networks where in 33- bus system the total active power of network is 3.7150MW and its reactive power is 2.30Mvar. The voltage of this system is also 12.66kV [25]. In 69- bus system, sum of active charge and its reactive charge are 3.8MW and 2.69MW, respectively. The voltage of buses in this system is 12.66kV [26]. In first system, the value of losses before installation of DG supply is 0.211MW and it is 3.8MW in the second system.

The costs of installed DG are also given in Table 1. The useful life for the project and DG has been considered 15 years. Likewise, whereas The given DGs are of wind and solar types thus fuel cost has been ignored. K- Coefficient is also equal to electricity sale cost that has been considered as 3718RS.

Table 1. Costs of wind and solar power systems

Fuel cost	Repairs and maintenance (Rs/MWH)	Primary investment (Million Rs/MW)	Type of DG supply
0	0.2	45	Wind
0	0.5	250	Solar

5- Simulations

a) Results of simulation for 33- bus system

The results of simulations on 33- bus system and for two wind and solar types of DG supply are given in Table 2. In Table 1 in which the costs of supplies have been listed, the cost of primary investment and repairs and maintenance cost of solar power system were higher than in wind type system and for this reason, it is also seen that the produced income from solar power system is lower than in wind power system. Bus no 19 with capacity of 3.4314MW has been identified for DG supply of wind type and also Bus no 6 with capacity of 2.5614MW has been determined for solar power system as the optimal location (siting) and capacity (sizing). Diagram of income based on bus- number and profile of the voltages before and after installation of supply with optimal capacity are shown in Figures 2,3,4 and 5 for wind and solar DG systems, respectively. As it observed, voltage profile has been improved for both types of supply.

Table 2. Results of simulation of 33-bus system

Target function (income) Million Rs	Optimal capacity MW	Number of optimal bus	Type of DG supply
88.758	3.4314	19	Wind
6.5593	2.5614	6	Solar

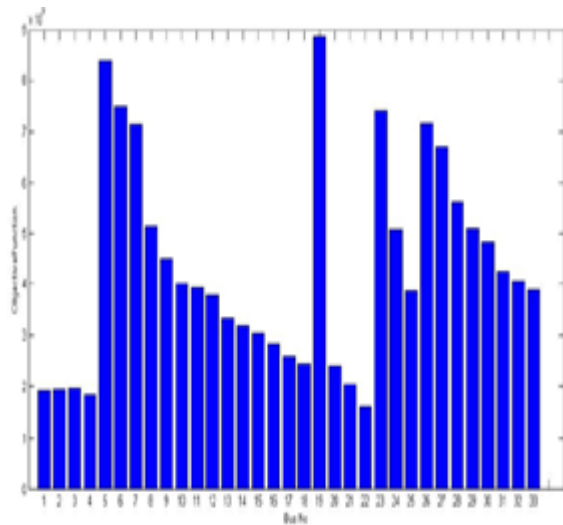


Figure 2. Diagram of income after installation of supply (of wind type)

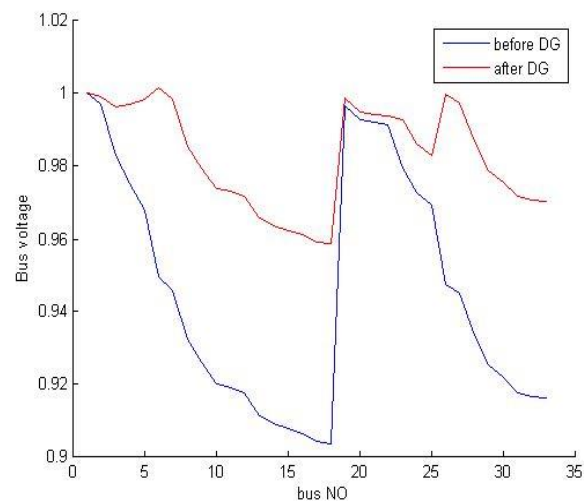


Figure 5. Diagram of voltage profile before and after installation of supply with optimal capacity of solar type

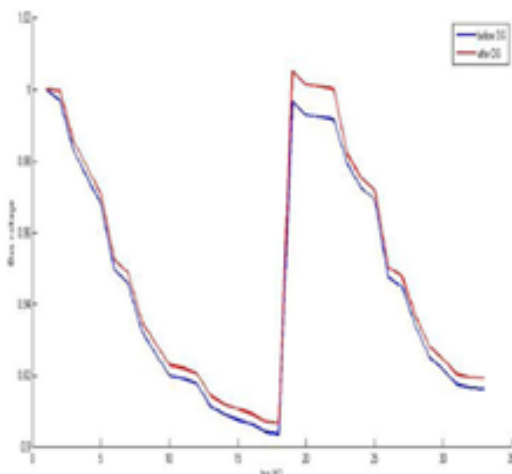


Figure 3. Diagram of voltage profile before and after installation of supply with optimal capacity of wind type

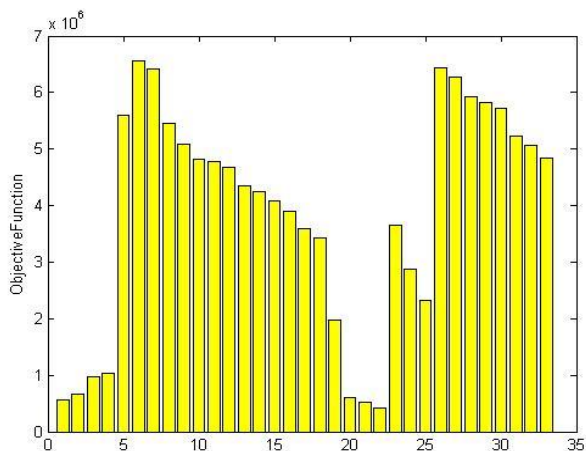


Figure 4. Diagram of income after installation of supply (of solar type)

b) Results of simulation for 69- bus system

The given results from the proposed method for 69- bus system are presented in Table 3. As it seen, the income obtained from wind type system is higher than in solar type system while the costs of investment, repairs, and maintenance in this system are lower than in solar type system. Similarly, according to the technique proposed in this system, the best location for installation of DG supply is obtained in wind- type system on bus-6 with its optimal capacity 3.8022MW. The best location for solar type system to acquire the maximum income is related to bus no 61 with capacity of 1.8768MW. The diagram of income based on bus number for DG supply of wind and solar types for this system has been also shown in Figures 6 and 7, respectively. Likewise, diagram of change in voltage profile after presence of wind and solar DG supply is indicated in Figures 8 and 9 as well, respectively. As it observed, the voltage profile has been improved in both types of supply.

Table 3. Results of simulation in 69- bus system

Target function (income)	Optimal capacity MW	Number of optimal bus	Type of DG supply
99.465	3.8022	6	Wind
7.9617	1.8767	61	Solar

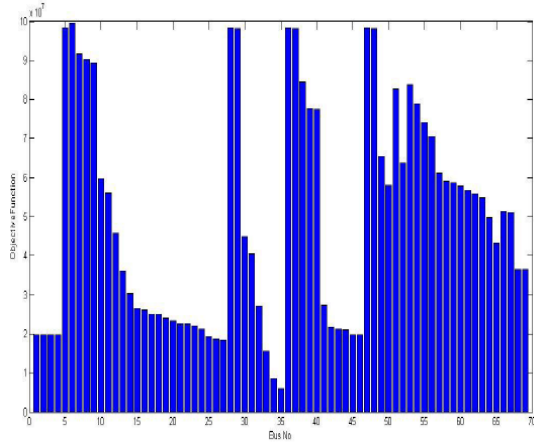


Figure 6. Diagram of income after installation of supply (of wind type)

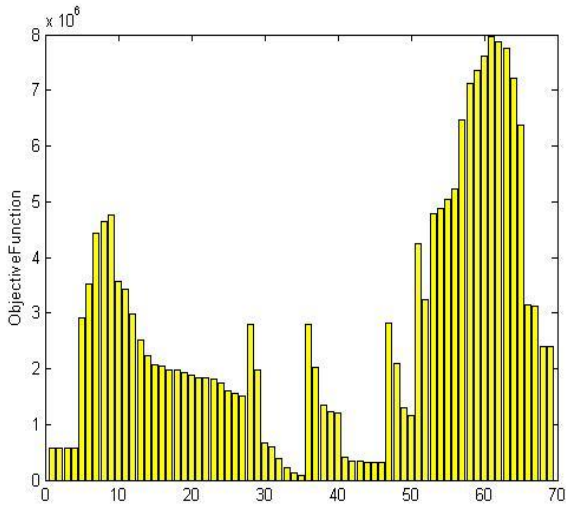


Figure7. Diagram of income before and after installation of supply with optimal capacity (of solar type)

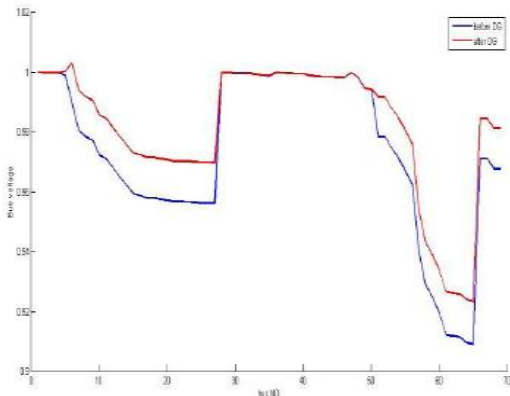


Figure 8. Diagram of voltage profile before and after installation of supply with optimal capacity (of wind type)

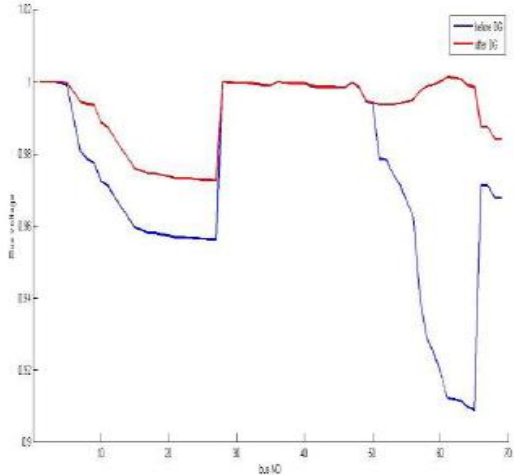


Figure 9. Diagram of voltage profile before and after installation of supply with optimal capacity (of solar type)

6- Conclusion

Determination of location (siting) and optimal capacity (sizing) of Distributed Generation (DG) supplies is considered as one of the important issues in design and installation of such supplies and several techniques have been proposed for this operation. In terms of design, presentation of a method may be useful in which both technical and economic aspects of this system are considered. An analytical method has been presented for siting and optimal sizing of DG for improvement of technical and economic improvement. The mathematical rules and principles have been employed in this technique to determine maximum income. Target function has been written as a function of DG supply power and then critical points of this function were determined by means of derivatives and the extremum (maximum point) of that function has been identified.

In this essay, target function includes technical and economic parameters comprising of profit due to sale of energy to network, profit returned from reducing of losses, cost of primary investment, fuel cost, cost of repairs and maintenance, and cost of replacement of equipments. Simulations have been implemented for both wind and solar types of DG supply. The studies have shown that the rate of income is higher in wind power system in which it has lower cost of primary investment than solar type. In addition, the effect of DG supplies on voltage profile as well and as we expected both types of supply have improved voltage profile. As it characterized from the results, the suggested method may be appropriate for siting and sizing for any type of DG supply. The offered technique can be adapted to synchronous siting of several DG supplies as well. We try to implement siting operation simultaneously for several DG supplies by developing the relations and applications of the suggested technique.

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