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Solution of boundary value problem for fuzzy fractional differential equations by using differential transform method (DTM)

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ABSTRACT

In this study, we implement a well-known transformation technique differential transform method (DTM), to the area of fractional differential equations, in this Paper, the solution to fuzzy fractional Initial Value Problem (FFIVP) under copout-type fuzzy fractional derivatives are define based on Hukuhara difference and strongly gene fuzzy differentiability. Also numerical examples are carried out for various types of problems including the Bagley-Torvik, Ricatti and composite fractional oscillation equations for the application of the method.

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Introduction

Fractional Calculus and fractional differential equations have undergone expanded study in recent years as a considerable in tersest both in mathematics and in applications. Recently, Agarwal etal [7]. Proposed the concept of solutions for fractional differential equations with uncertainty. They con side red. Riemann Ville differentiability to solve FFDES. In this paper, we propose Riemann-liouville differentiability by using Hukuhara difference so-called Riemann-liouville H-differentiability. [7, 13], the successful application of fractional differential Equations (FDES) in mo de ling such as viscose lastic material [1], control [2], signal processing [3] and etc.

$$(I_{a+}^{\beta})f(t) \quad = \frac{1}{\Gamma(1-\beta)}\int_a^t \frac{f(s)}{(t-s)^{\beta}}ds$$
$$= \frac{1}{\Gamma(\beta)}\int_a^t (t-s)^{\beta-1}f(s)ds, x > a, \beta \in (0,1] \qquad (1)$$

note also that when $f \in c([0,\infty) * E, E)$ Then $I_c^{\beta} f \in c([0,\infty) * E, E)$

Definition 1 *The corresponding nonlinear ,Appropriate initial condition for*

$$(D_C^{\alpha})f(t) - \lambda G(t, f(t)) = \sigma(t)$$
(2)

Let $\alpha \in (0,1-\lambda) \in R$, $\sigma \in ([0,T) * E, E), T > 0$

We consider the following fuzzy linear fractional differential equation

$$\lim_{t\to 0+} t^{1-\alpha} f(t) = f_0$$
(3)

where $G: S = \{(t, f(t)), f(t) \in (0,\infty)\} \to E$ Consider the space E of fuzzy sunsets of E^n, with the metric.

$$d_{\infty}(u,v) = supd_H([u]^{\alpha},[v]^{\alpha}), u,v \in E. \qquad (4)$$

Definition 2 *Let* $\alpha, \beta, > 0$ *, the Mittag -Leffler function is definition*

$$E_{\alpha,\beta}(t) = \Sigma_{k=0}^{\infty} \frac{t^k}{\Gamma(\alpha k+\beta)}, t \in C, \alpha, \beta \neq 0$$

$$E_{\alpha}(t) = \Sigma_{k=0}^{\infty} \frac{t^k}{\Gamma(\alpha k+1)}, t \in C,$$

For $\alpha > 0$ and $x \in R, x > 0$, it is clear that $E_{\alpha,\beta}(t) > \frac{1}{\Gamma(\alpha)}$ for $x < 0$, showing that $E_{\alpha,\beta}(t) > o$, is not so trivial But asian the feat that $E_{\alpha}(-t), t \in (0,\infty)$ is completely monodic, we have that $E_{\alpha,\beta}(t) > 0$.

Definition 3 *[3] et* $f: [a,b] \to E$ *,the fuzzy Riemann-Liouville integral of fuzzy-valued function f is defined as follows:*

A function $F: [a,b] \to E$ is differentiable at a point $t_0 \in (a,b)$, if there is such an element $F'(t_0) \in E$, that the Limits

$$\lim_{t\to 0^+} \frac{F(t_0+h)-F(t_0)}{h} = \lim_{t\to 0^+} \frac{F(t_0)-F(t_0-h)}{h},$$
(6)

$$\lim_{t\to 0^-}\frac{F(t_0+h)-F(t_0)}{h}=\lim_{t\to 0^-}\frac{F(t_0)-F(t_0-h)}{h},$$
(7)

Definition 4 *Let f is Continuous fuzzy-value function suppose that F inverse transform*

$$f(t)=\sum_{k=0}^{k=\infty}F(k)(t-t_0)^k, f(t,r)=[f_-(t,r),f^-(t,r)]$$
(8)

where

$$f_-(t,r)=\sum_{k=0}^{k=\infty}\frac{d^k f_-(t)(t-t_0)^k}{k!dt^k}, f^-(t,r)=\sum_{k=0}^{k=\infty}\frac{d^k f^-(t)(t-t_0)^k}{k!dt^k}.$$
(9)

Definition 5 *A function $F:[a,b]\to E$ is differentiable at a point $t_0\in(a,b)$, Let $f\in C^n[a,b]*L^n[a,b]$ is a fuzzy-value function the fuzzy caupto integral of a fuzzy valued function F con be expressed as follows*

$$(D_C^\alpha)f(t,r)=I^{n-\alpha}(f^{(n)})^-(t,r)$$
(10)

$$(I^{n-\alpha})f^{(n)}(t,r)=[(I^{n-\alpha})f_-^{(n)}(t,r),(I^{n-\alpha})(f^-)^{(n)}(t,r)].$$
(11)

2 Generalized Taylor's formula

Geralized tailor's formula under the caputo-type fractional derivative was introduced in crisp context [2] Here we introduced it under the coputo-type fuzzy fractional derivatives as follows:

Theorem 1 *Let $F:[a,b]\to E$ and $f\in C^n[a,b]*L^n[a,b]$ is $[i-\alpha]$ differentiable and suppose that $I_c^\beta f\in c([0,\infty)*E,E)$, where Then we have $t_0\in(0,t)$ and $t\in[0,T)$*

$$f(t)_\alpha=[f_-(t,\alpha),f^-(t,\alpha)],\alpha\in[0,1]$$
(12)

$$[f_-(t,\alpha)]=\sum_{i=0}^{i=n}\frac{D_C^{i\alpha}f_-(0)t^{i\alpha}}{\Gamma(i\alpha+1)}+\frac{D_C^{(n+1)\alpha}f_-(t_0)t^{(n+1)\alpha}}{\Gamma(n\alpha+\alpha+1)}$$
(13)

$$[f^-(t,\alpha)]=\sum_{i=0}^{i=n}\frac{D_C^{i\alpha}f^-(0)t^{i\alpha}}{\Gamma(i\alpha+1)}+\frac{D_C^{(n+1)\alpha}f^-(t_0)t^{(n+1)\alpha}}{\Gamma(n\alpha+\alpha+1)}$$
(14)

where

$$D_C^{i\alpha}f^-(0)=D_C^{i\alpha}f^-(t_0)|_{t=0}$$
(15)

$$D_C^{i\alpha}f_-(0)=D_C^{i\alpha}f_-(t_0)|_{t=0}$$
(16)

Proof. Using the relation of

$$(I_C^\alpha D_C^\alpha)f_-(\alpha)=f_-(\alpha)-\sum_{k=0}^{k=n}\frac{t^k f_0^{\alpha k}}{k!}$$

and $(I_C^{n\alpha}D_C^{n\alpha})f_-(\alpha)-(I_C^{(n+1)\alpha}D_C^{(n+1)\alpha})f_-(\alpha)=\sum_{i=0}^{i=n}\frac{t^{i\alpha}f_0^{i\alpha}}{\Gamma(i\alpha+1)}$

Applying the integral mean value theorem to (3)

Theorem 2 *Let $F:[a,b]\to E$ and $f\in C^n[a,b]*L^n[a,b]$ is $[i-\alpha]$ differentiable and suppose that $I_c^\beta f\in c([0,\infty)*E,E)$,where Then we have $t_0\in(0,t)$ and $t\in[0,T)$ then have*

$$f(t,\alpha)=f(t_0,\alpha)+\frac{1}{\Gamma(\alpha)}D_C^\alpha f(\tau,\alpha)(t-t_0)^\alpha,\tau\in[t_0,t] \quad (17)$$

and $D_C^\alpha$is caputo fuzzy fractional derivatives of order $\alpha>0$.

Proof. from the Rieman- liuville fractional derivative of order $\alpha\in(0,1]$ of a continues function $F:[a,b]\to E$

$$D_C^{i\alpha}f(t)=\frac{1}{\Gamma(1-\alpha)}\frac{d}{dt}\int_0^t(t-s)^{-\alpha}f(s)ds=\frac{d}{dt}I^{1-\alpha}f(t) \quad (18)$$

$$D_C^{i\alpha}f(t)=f(t,\alpha)-\sum_{k=0}^{k=m-1}\frac{f^{(k)}(t,\alpha)}{k!}(t-t_0)^k$$
(19)

Using the integral mean value theorem we get

$$I_C^{i\alpha}D_C^{i\alpha}f(t,\alpha)=f(t,\alpha)-\sum_{k=0}^{k=m-1}\frac{f^{(k)}(t,\alpha)}{k!}(t-t_0)^k=\frac{1}{\Gamma(\alpha)}D_C^{i\alpha}f(\tau,\alpha)(t-t_0)^\alpha,\tau\in[t_0,t]$$
(20)

for we have $\tau\in[0,t]$, $I_C^\alpha D_C^\alpha f(t,\alpha)=f(t,\alpha)-f(t_0,\alpha)$ then $I_C^\alpha D_C^\alpha f(t,\alpha)>0$

3 Solving fuzzy initial value problem of fractional

In this section, the generalized differential trans form method for solving fuzzy initial value problem of fractional order under the coputo-type fuzzy fractional derivative will be presented. The method is based on the fractional differential transform method-used as a prediction at each step-and the Modified trapezoidal rule-used to make a correction to obtain the finite value at each step-which hues been proposed by o dib at and momani [5] Consider the following FFIVP

$$(D_C^\alpha)f(t)-\lambda G(t,f(t))=\sigma(t)$$
(21)

Let $\alpha\in(0,1-\lambda)\in R$, $\sigma\in([0,T)*E,E)$, $T>0$ We consider the following fuzzy linear fractional differential equation

$$\lim_{t\to 0+}t^{1-\alpha}f(t)=f_0$$
(22)

4 Factional differential transform method

In this section, we are going to define fuzzy differential trans from for fuzzy-valve function more over, we will consider the properties of the fuzzy differential transforms, then a derivative definition(2), and (1) is given order to connect between differential transforms of caputo fractional derivative and corresponding fuzzy-value fumet ion. Let us expand the analytical and continuous function $f(t)$ in terms of a fractional power series as follows:

$$f(t)=\sum_{k=0}^{k=\infty} F(k)(t-t_0)^k \tag{23}$$

Where is the order of function and $F(k)$ is the fractional differential transform of $f(t)$the definition in(5) should be modified to deal with integral ordered initial conditions in caputo sense [5] as follows

$$D_C^\alpha\left(f(t)-\sum_{k=0}^{k=m-1}\frac{f^{(k)}(0)(t-t_0)^k}{k!}\right)=\frac{1}{\Gamma(n-\alpha)}\frac{d^n}{dt^n}\int_0^t\frac{\left(f(t)-\sum_{k=0}^{k=m-1}f^{(k)}(0)(t-t_0)^k\right.}{(t-x)^{1+\alpha-n}}dt \tag{24}$$

The transformations of the initial conditions are defined as follows

$$IF\,\frac{K}{\alpha}\in Z^+ then\,\frac{1}{(\frac{k}{\alpha})!}\left[\frac{d^{\frac{k}{\alpha}}f(t)}{dt^{\frac{k}{\alpha}}}\right] \tag{25}$$

Theorem 3 *If* $[f(t)]_\alpha = D_C^\alpha[g(t)]_\alpha$ *then*

$$F(K)=\frac{\Gamma(q+1+k)}{\Gamma(1+\frac{k}{)}\alpha}G(K+\alpha q)$$

Proof. For $f:(a,b)*E\rightarrow E$ and $g:(a,b)\rightarrow E$ the caputo sense fractional different ion of $f(t)$ con be written as follows:

$$D_C^\alpha\left(g(t)-\sum_{k=0}^{k=m-1}\frac{g^{(k)}(0)(t-t_0)^k}{k!}\right)=\frac{1}{\Gamma(n-\alpha)}\frac{d^n}{dt^n}\int_0^t\frac{\left(g(t)-\sum_{k=0}^{k=m-1}g^{(k)}(0)(t-t_0)^k\right.}{(t-x)^{1+\alpha-n}}dt \tag{26}$$

$$D_C^\alpha(g(t))=\frac{1}{\Gamma(n-\alpha)}\frac{d^n}{dt^n}\int_0^t\frac{\sum_{k=0}^{k=m-1}(t-t_0)^k(G(K))-\sum_{k=0}^{k=m-1}g^{(k)}(0)(t-t_0)^k}{(t-x)^{1+\alpha-n}}dt \tag{27}$$

$$D_C^\alpha(g(t))=\sum_{K=0}^{K=q\alpha}\frac{\Gamma(q+1+\frac{k}{\alpha})}{\Gamma(1+\frac{k}{\alpha})}G(K)(t-t_0)^{\frac{k}{\alpha}} \tag{28}$$

From the definition of transform in Eq (3), the following expression is obtained.

$$F(K)=\frac{\Gamma(q+1+\frac{k}{\alpha})}{\Gamma(1+\frac{k}{\alpha})}G(K+\alpha q)$$

(29)

$y(k)$ is evaluated of to certain number of terms and then using the inverse transformation rule,$y(t)$ is evaluated as follows

5 Conclusion

In this work, we carefully developed a new generalization of the differential transform method that will extend the application of the method to efferential equations of fractional order. The new generalization avoids the error made in [1] by using the generalized Taylor's formula and Caputo fractional derivative.

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