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A New Mathematical Model for a Bi-objective Location Routing Problem Solved by A Multi-Objective Genetic Algorithm

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ABSTRACT

Location of warehouses and routing of vehicles are two essential key points in order to distribute perishable products properly. Poor location-routing tasks may cause tremendous losses. In this paper, a bi-objective mixed integer mathematical programming is proposed to reduce the total cost of the supply chain and to balance the workload of distribution centers, concurrently. A multi-objective evolutionary algorithm called Non-Dominated Genetic Algorithm-II (NSGA-II) is customized to generate set of non-dominated solutions on Pareto front of the problem. The performance of proposed algorithm and an efficient exact multi-objective method, called ε-constraint, is compared on several benchmark instances using several performance measurements. The analysis reveals the efficacy and applicability of proposed method.

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1.Introduction

Proper transportation system plays a main role in distribution of products with low cost, short time, and high quality. Supplying perishable products causes some challenges in the chain. The trade-off between time and cost of supply is an essential problem in such supply chains and required adequate planning concerns location of warehouses and routing of transportation vehicles. The most important factor is the time of distribution, because the products are decayed very soon. In this concern the location of warehouses and manufacturer are became very important issues. Nearness of manufacturing and depots to the supply places (i.e., customers, and markets) reduces the time of supply but the total cost of the supply chain may increase dramatically. All these are affected by proper location of distribution centers and also right routing of transportation vehicles. Improper location of distribution centers may cause difficulties in routing of transportation vehicles as well as unbalanced workload of distribution centers. As the total cost of transportation is a main factor so, a trade-off is formed. In this paper, this issue is resolved and revisited through the location of distribution centers and the possibility of transportation between distribution centers and customers.

Delivery of final product to customers through a supply chain involves several transportation activities among segments of chain. It should be noted that the segments of chain may be far from each other geographically. A major part of logistic cost is related to the transportation cost. (Alumur and Kara, 2007) [2]. One of the problems in logistic management is location-routing. The main aim in location-routing is to determine the number and location of facilities and also the optimum route for transportation vehicles. Webb, (1968) and Christofides and Elion, (1969) found that it is not right to count the cost of products delivery by considering direct transportation between depot and customers. Nowadays, satisfaction of customers is reliant on the reduction of the distribution time (Bramel and Simchi-Levi, 1997) [5]. So, decision making about the location of depots and the routes of distribution is of great importance and can affect the operation of the whole chain. Salhi and Rand,discussed the interactive relation between Location-Routing and also the consequences of ignoring issues related to route finding. The issue of location-routing is closely related to the Location-Allocation and vehicle routing problems (VRP) [19]. In fact, Location-Allocation and VRP are special cases of Location-Routing. Compared to the Location problems, the problems of Location-Routing are more difficult. In the location problems, distribution points are located at the center of demand points while in location-routing problems, distribution points should be located at the center of those demand points which are met by the associated route.

2. Literature

Since both mentioned problems are NP-Hard, the problem of Location-Routing is also NP-Hard (Nagi and Salhi, [15]). Dantzig and Ramser, presented a precise solution to the classical model VRP [5]. Clarke and Wright, presented an economizing algorithm for the VRP [8]. Fisher et al., developed various approaches to solve the VRP [11]. Meta-Heuristic methods, such as Genetic Algorithm (GA), Tabu Search (TS), and Ants Optimization System (ACO) have been proposed for VRP [9]. Potvin and Gendreau, proposed an exact multi-objective method, called ε-constraint method, in order to solve a bi-objective Traveling Salesman Problem (TSP) [17]. Banos and Ortega, discussed a multi-objective VRP in which tried to minimize the cost of transportation system as well as the distances covered by transportation vehicles, concurrently [4]. The latter objective is called distance balance.Ahmadi-Javid and Seddighi, [11] proposed a location-routing problem in a supply-chain network with a set of producer–distributors that produced a single commodity and distributed it to a set of customers. The production capacity of each producer–distributor varied randomly due to a variety of possible disruptions, and the vehicles involved in the distribution system were disrupted randomly. Najafi et al., proposed a multi-objective, multi-mode, multi-commodity, and multi-period stochastic model to manage the logistics of both commodities and injured people in the earthquake response [16]. Also, a robust approach was developed and used to make sure that the distribution plan performed well under the various situations. Meisel et al., presented a model and solution approach for combining production and intermodal transportation planning in a supply network [13]. The model included relevant decisions regarding production setups and output volumes of plants, cargo consolidation at intermodal terminals, and capacity bookings for road and rail transports. A Branch-and-Cut method and heuristics were designed to handle the problem .

Mohammadi et al., proposed a novel sustainable hub location problem (SHLP) in which two new environmental-based cost functions accounting for air and noise pollution of vehicles were incorporated [14]. Mohammadi et al., proposed a mixed possibilistic–stochastic programming approach to handle the uncertainty [14]. A simulated annealing (SA) and an imperialist competitive algorithm (ICA) with a new solution representation were developed to solve real-sized instances. Some computational experiments were provided to demonstrate the effectiveness of the proposed model and solution approaches.

On time delivery of perishable products is the most important factor in the process of distribution which is affected by location of distribution centers and facilities. Moreover, the cost of distribution is another factor plays essential role in this area. The problem involves in both cost reduction and on time delivery. The cost and time of distribution are closely related to the location of warehouse facilities and distribution centers as well as routing of transportation vehicles. In real cases, poorly location or routing may cause additional cost and unbalanced work load of warehouses. The aforementioned factors have rarely been considered in literature, concurrently. In this paper, a multi-objective mathematical programming is proposed in order model a location-routing problem for distribution of perishable products. Two types of objective functions, i.e., total cost of system and balanced work load of distribution centers are considered, simultaneously. Then, an evolutionary computation algorithm, called NSGA-II, is developed to solve the proposed multi-objective mathematical programming. The performance of NSGA-II algorithm is compared with an exact method, called epsilon-constraint, using several multi-objective metrics .

2.1 Multi-Objective Location-Routing Problem for Distribution of Perishable Products

The following assumptions are considered in order to model the multi-objective location-routing problem for distribution of perishable products.

- The positions, numbers, and demands of customers are fixed during planning period.
- The positions, numbers, and costs of potential warehouses are fixed during planning period.
- Types and the capacities of transportation vehicles are predetermined.
- Each vehicle is belonged to one distribution center.
- The location of producer is fix during planning period.
- There is no capacity limitation for production.
- The parameters are not changes during planning period.
- Demand of all customers must be delivered during planning period.
- Demand of a customer must be satisfied through one supplier.
- A route is assigned to only one transportation vehicle.
- A route is began from one distribution center and ended at the same center.
- There is a predetermined maximum allowable time for delivery of a product to each customer.
- The maximum coverage distance of transportation vehicles is definite.
- The velocity of transportation vehicles and consequently the time of transportation are predetermined and fixed during planning period.

The problem is seeking to make optimum decisions about the location of distribution centers, the number of distribution centers, and the number of equipment in each distribution center. Also, problem seeks to find optimum allocation of customers to distribution centers and to determine the optimum routes of transportation vehicles in a way that the total cost of the system are minimized while the workload of transportation in distribution centers is balanced.

Notations

The sets, parameters, and decision variables used in this research are defined as follows.

Sets

I	set of potential depot locations (i=1,2,…, m)
J	set of customers (j=1,2,…, n)
K	set of vehicles

2.1.1 Model parameters

- O_i Fixed cost of establishing a depot at site i,
- d_j Demand of customer j,
- α_i Variable cost per unit throughput at depot i,
- P_k Unit cost of transportation by vehicle k,
- q_{ij} Fixed cost of sending product from depot i to customer j,
- V_i Maximum throughput of depot i,
- C_{ij} distance from point i to point j (i, j ∈ I ∪ J),
- t_{ijk} Traveling time from point i to point j by vehicle k (i, j ∈(I ∪ J),k ∈ K),
- τ_{kj} Maximum time to unload demands of customer j by vehicle k (j ∈ J, k ∈ K),
- E_k Maximum allowable length of route for vehicle k,
- D_k Capacity of vehicle k,
- T Maximum allowable duration of distribution time.

2.1.2 Decision variables

- x_{ijk} It is equal to 1, if point j is immediately met after point i by vehicle k (i, j ∈ I ∪ J, k ∈ K); otherwise0.
- y_i It is equal to 1, if a depot is located at site i; otherwise 0.
- z_{ij} It is equal to 1, if customer j is served from depot i; otherwise 0.
- u_{ik} Auxiliary variables used in sub-tour elimination constraints.

2.1.3 Mathematical Model

The Model (1)-(16) is proposed for multi-objective location-routing problem in order to distribute the perishable products.

$$Min\ f_1 = \sum_{i\in I} O_i y_i + \sum_{i\in I}\sum_{j\in J}(\alpha_i d_i + q_{ij}) z_{ij} + \sum_{k\in K}\sum_{i\in(I\cup J)}\sum_{j\in(I\cup J)} P_k C_{ij} x_{ijk} \quad (1)$$

$$Min\ f_2 = argmax_{i\in I}\left\{\sum_{k\in K}\sum_{j\in(I\cup J)} P_k C_{ij} x_{ijk}\right\} - argmax_{i\in I}\left\{\sum_{k\in K}\sum_{j\in(I\cup J)} P_k C_{ij} x_{ijk}\right\} \quad (2)$$

Subject To:

$$\sum_{k\in K}\sum_{i\in(I\cup J)} x_{ijk} = 1 \quad \forall j \in J \quad (3)$$

$$\sum_{i\in(I\cup J)}\sum_{j\in J} d_j x_{ijk} \le D_k \quad \forall k \in K \quad (4)$$

$$\sum_{i\in(I\cup J)}\sum_{j\in(I\cup J)} C_{ij} x_{ijk} \le E_k \quad \forall k \in K \quad (5)$$

$$\sum_{j\in J}\sum_{i\in(I\cup J)} \tau_{kj} x_{ijk} + \sum_{i\in(I\cup J)}\sum_{j\in(I\cup J)} t_{ijk} x_{ijk} \le T \quad \forall k \in K \quad (6)$$

$$\sum_{i\in I}\sum_{j\in J} x_{ijk} \le 1 \quad \forall k \in K \quad (7)$$

$$\sum_{j\in(I\cup J)} x_{ijk} - \sum_{j\in(I\cup J)} x_{ijk} = 0 \quad \forall k \in K, i \in (I\cup J) \quad (8)$$

$$\sum_{j\in J} d_j z_{ij} \le V_i y_i \ \forall i \in I \quad (9)$$

$$-z_{ij} + \sum_{u\in(I\cup J)} (x_{iuk} + x_{ujk}) \le 1 \ \forall i \in I, \forall j \in J, \forall k \in K \quad (10)$$

$$\sum_{j\in J} z_{ij} \ge y_i \quad \forall i \in I \quad (11)$$

$$u_{ijk} - u_{jk} + N x_{ijk} \le N - 1 \quad \forall i,j \in (I\cup J), k \in K \quad (12)$$

$$x_{ijk} \in \{0,1\} \quad \forall i \in (I\cup J), j \in (I\cup J), k \in K \quad (13)$$

$$y_i \in \{0,1\} \quad \forall i \in I \quad (14)$$

$$z_{ij} \in \{0,1\} \quad \forall i \in I, \forall j \in J \quad (15)$$

$$u_{ik} \in integer^+ \quad \forall i \in I, k \in K \quad (16)$$

The objective function (1) minimizes the total cost of the system, including the cost of establishing distribution centers, delivering products to the customers, and transportation cost. The objective function (2)tries to minimize the difference between maximum and minimum transportation cost of distribution centers. On the other hand the objective function (2) balances the workload of distribution centers. The set of constraint (3), which is written for each customer, guaranties that each consumer is served by only one transportation vehicle. The set of constraint (4), which is written for each vehicle, assures that all products transported by a certain vehicle is less than or equal to its capacity. The set of constraint (5), which is written for each vehicle, guaranties that the maximum allowed length of route for each vehicle is satisfied. The set of constraint (6), which is written for each vehicle, assures that transportation time of a vehicle is less than or equal to a predetermined value. The set of constraint (7), which is written for each vehicle, shows that each transportation vehicle can at most be dispatched one time. The set of constraint (8) assures that a given vehicle departed a customer that had served. The set of constraint (9), which is written for all depots, assures the capacity of warehouse. The set of constraint (10) shows that in a route including both distribution center and customers, the demand is allocated to the customers. The set of constraint (11) guarantees that a distribution center can serve the customers if and only if it is established. The set of constraint (12) assures the sub-tour elimination. The set of constraints (13)-(15) represent the type of decision variables.

3.Solution Approaches

We have proposed two multi-objective solution approaches, including and exact method and an evolutionary computation method, for the proposed Model (1)-(16). Both of them generated the Pareto front of the instances of the problem and the performances have been compared using multi-objective metrics on accuracy and diversity of re-generated Pareto front. In this part, first the exact method, called epsilon-constraint is proposed. Then, the evolutionary computation algorithm, called NSGA-II is presented.

3.1 Epsilon-constraint method

A general form of Multi-Objective Decision Making (MODM) problem is shown as (17).

$$Max(f_1(x), f_2(x), \dots, f_p(x))$$
$$\text{S.T.} \quad X \in S \quad (17)$$

Using the epsilon-constraint method, the MODM problem (17) is converted into Model (18).

$$Max\ f_1(x)$$

S.T.

$$\begin{aligned} & X \in S \\ & f_2(x) \geq e_2 \\ & f_3(x) \geq e_3 \\ & \vdots \\ & f_p(x) \geq e_p \end{aligned} \tag{18}$$

By parametrical variation in the RHS of the constrained objective functions (e_i) the efficient solutions of the problem are obtained. On the other hand, the right hand sides of constrained objective functions ($\mathrm{i.e.}, \mathrm{e_i}$) are changed in a feasible range from minimum to maximum value for associated objective function. These minimum and maximum values are easily calculated based on single objective optimization problem.. Selection of the most important objective function is done by decision maker (Branke et al., [6]).

4.Proposed Customized NSGA-II algorithm

NSGA-II is one of most well-known multiple-objective evolutionary computation algorithms. The stages of this algorithm are similar to the simple genetic algorithm (SGA). The only difference between SGA and NSGA-II is that in SGA, the best possible answer is to be found while a set of non-dominated solutions, called Pareto front, are to be generated in NSGA-II(Deb et al., [10]).

The concept of domination is one of the central concepts in multiple-objective optimization procedures. This concept is taken as the main criterion for comparing the generated answers in each iteration of the algorithm. It is said that $x^{(1)}$dominates $x^{(2)}$if and only if in all objective functions, $x^{(1)}$ is not worse than $x^{(2)}$; second, at least in one of the objective functions, $x^{(1)}$ is absolutely better than $x^{(2)}$. It is said that $x^{(1)}$ strongly dominates $x^{(2)}$ when its value in all objectives is better than the value of $x^{(2)}$. If neither $x^{(1)}$ nor $x^{(2)}$do not dominate the other, it is said that both answers are non-dominated. Among a set of P answers, the non-dominated solutions are those which are not dominated by any P members. If P set constitutes the whole space in which the answers are seeking, the set of obtained non-dominated answers is called Pareto optimal set (real Pareto front). In real life problems, the real Pareto front cannot be achieved, so the estimation or real Pareto front is re-generated.

Two properties should be held for re-generated Pareto front: 1) accuracy, which is supplied through Pareto ranking and selecting individuals based on their ranks; and 2) diversity which is provided by a diversity measurement, called crowding distance. The selection is accomplished based on a binary tournament procedure among from highest ranked individuals till the archive size is reached. Ties are resolved using crowding distance measurement. Those points are ranked in a certain rank are ordered using crowding distance. For individuals in a certain rank as the crowding distance is higher the priority of selection is more.

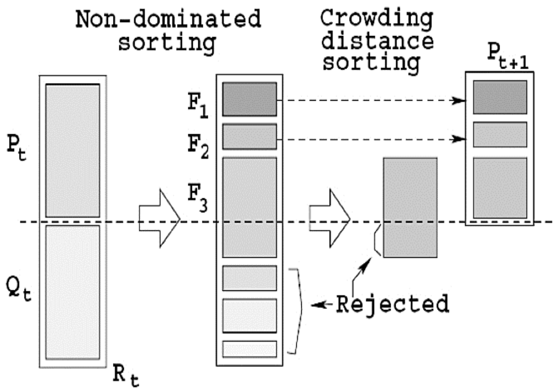

Figure 1: Schematic View of NSGA-II Procedure

As the NSGA-II has been widely discussed in literature (Deb et al., [10]), details are prevented to be described for sake of brevity. Figure 1 shows the schematic view of NSGA-II algorithm. As mentioned crowding distance is measurement for determining the diversity of solutions. Figure 2 presents the schematic view of crowding distance. We introduce the main customization which has been accomplished in this research in order to solve the location-routing problem.

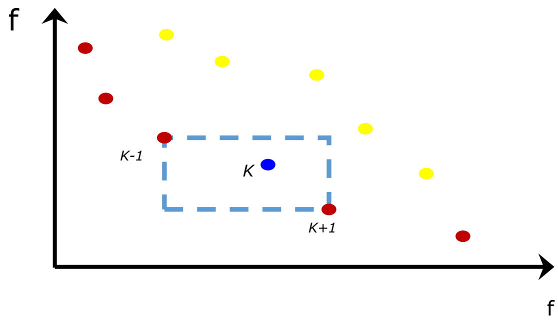

Figure 2: Crowding Distance for Solution *k* in a Two-Dimensional Objective Space

5.Representation of a Chromosome

Suppose that there are m*(i=1,…,m)* customers, k *(k=1,…,K)*transportation vehicles, and n *(n=1,…,N)* candidate point for establishing distribution centers. By these assumptions, a chromosome with m+2k locus is proposed. Three parts are considered in the proposed chromosome. The number of genes of first part is equal to k and its alleles are integer numbers between 1 and m. It shows that k^{th}transportation vehicle is assigned to i^{th}customer. The second part of the chromosome includes k genes and its alleles are non-repeated integer numbers between 1 and m. It shows the first customer that is served by k^{th}transportation vehicle. The third part of chromosome also includes m genes and its alleles are non-repeated integer numbers between 1 and m. It shows the order of offering services to the customers who are at the same route. Figure 3 presents coding and de-coding of a possible solution for the problem.

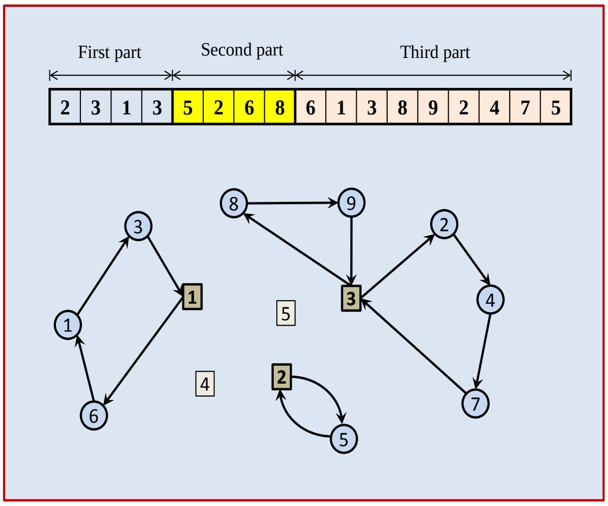

Figure 3: Coding and De-coding of a Solution

In Figure 3, there are five (n=5) candidate points for establishing distribution centers marked by gray squares. Nine customers are considered which are marked using blue circles. Four transportation vehicles are considered in Figure 3.

6.Crossover operator

A double-point crossover operator is used based on a crossover probability in the proposed algorithm. As the proposed chromosome has three sections, so some different conditions may appear as follows:

- Both cross points occur in the first section of chromosome; in other words among genes which are between 1 and k, or one of the cross points occurs after k^{th} gene and the other point occurs after $2k^{th}$ gene. In this condition, the operator is in normal condition and the middle part of the first parent and the first and third part of the second parent create the first child. The second child is constructed using the middle part of the second parent and the first and third part of the first parent.
- Both cross points occur in the second part of the chromosome; in other words, among genes which are between k+1 and 2k. In this case, the partially mapped crossover (PMX) operator is used. Firstly, the first and third parts of the second parent and the middle part of the first parent are combined to create the first child. Then, repeated alleles are replaced based on PMX operator. The second child is similarly formed. Figure 4 presents PMX crossover operator.

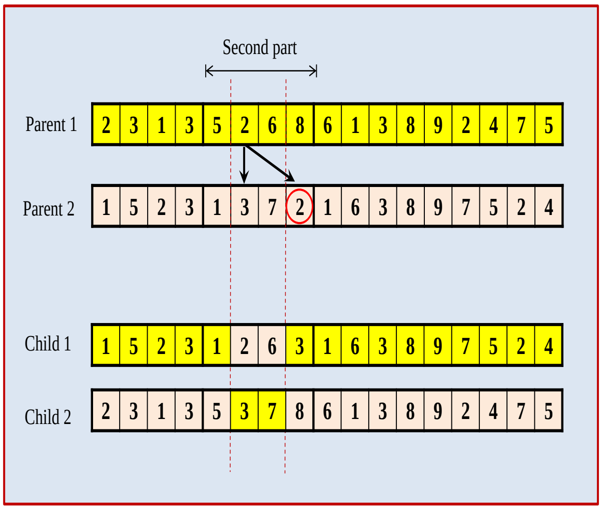

Figure 4: PMX Crossover (Cross Points in Second Part of Chromosome)

- Both cross points occur in the third part of the chromosome; in other words, among genes which are between 2k+1 and 2k+m. In this condition, the order crossover (OX) operator is used. The middle part of the first parent are found and determined among the genes of the second parent which are between (2k+1) and (2k+m). The other genes of the second parent are entered into the genes of the first and third parts. The second child is similarly formed. Figure 5 presents this OX crossover operator.

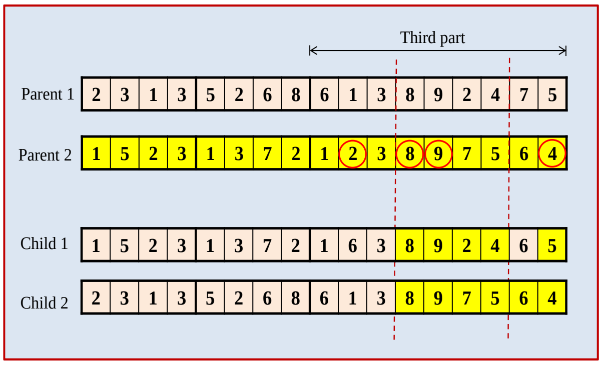

Figure 5: OX Crossover (Cross Points in Third Part of Chromosome)

- One cross point in the first part and the other is in the second part chromosome. In this case, the middle part of the first parent and the first part of the second parent are placed in first child. Also, the third part of the second parent is placed in the first child using PMX operator. The second child is reproduced in the same way.
- One cross point in the first part and the other is in the third part chromosome. In this condition, the middle part of the first parent, the first part of the second parent, and the third part of the second parent are placed in the first child using OX. The second child is also reproduced similarly.
- One cross point in the second part and the other is in the third part chromosome. In this case, the middle part of the first parent and the first part of

the second parent are placed in the first child using PMX. Also, the third part of the second parent is placed in the first child using OX.

7.Mutation operator

The mutation operator in this research is accomplished based on a mutation rate which is a parameter of the algorithm. Three cases may occur as follows:

- The selected gene for mutation is in the first part of the chromosome. In other words, it is between 1 and k. In this condition, the allele of the gene is randomly re-numbered using the eligible values (i.e., in this case among from the potential distribution centers). The same value in the first part of the chromosome is eligible. Figure 6 illustrates this case.

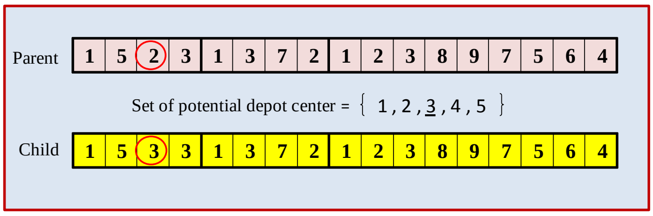

Figure 6: Mutation (mutation point in the first part of chromosome)

- The selected gene for mutation is in the second part of the chromosome. In other words, it is between k+1 and 2k. In this condition, the selected gene is randomly re-numbered among from the index of customers which have not been selected in second part of this chromosome as in the second part of the chromosome same value of allele is not eligible.
- Condition 3: The selected gene is in the third part of the chromosome. In other words, it is between 2k+1 and 2k+m. In this case, in addition to the selected gene, another gene from the third part of the chromosome is randomly selected. Then, by using swap mutation operator, the alleles of these genes are changed. Figure 7 illustrates this case.

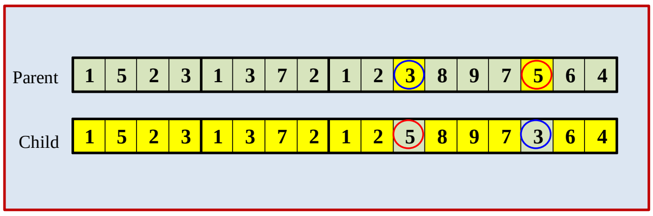

Figure 7: Mutation (mutation point in the third part of chromosome)

8.Results

In this section, the ways in which the parameters are set and also the results obtained from calculations test problems are presented. Several test problems in small, medium, and large size are used to test the performance of proposed NSGA-II for location-routing of perishable product problem. In order to show the efficiency of the NSGA-II algorithm, ten problems in small and medium sizes were designed and the results obtained from this method were compared with the results obtained from epsilon-constraint method. Then, the efficiency of both methods have been compared using several multi-objective metrics. Finally, the result of NSGA-II has been reported for large size instances, as the epsilon-constraint is unable to solve the large size instances.

8.1 Setting the parameters

The efficiency of an algorithm is heavily related to its parameters. Different parameters might produce different quality of answers. If the parameters are not set appropriately, high quality answers cannot be obtained. In order to set the parameters for the proposed algorithm of this study, we have tested several settings. In order to do this, one problem is selected randomly and by changing the parameters, the problem is solved till the best setting is reached. Table 1 shows the selected parameters of NSGA-II.

Table 1: Parameters of NSGA-II

Parameter	Value
Population size	50
Crossover rate	0.8
Mutation rate	.05
Maximum Iteration	200

In epsilon constraint method, the range of minor objective function has been divided into 10 pieces. In each run the right hand side ofminor objective functions, which has been a constraint, is set equal to one of the break points. In this way, the optimum value of main objective function is obtained while the minor objective function is equal to a feasible solution. This will cause generating non-dominated solutions. Iterating this procedure will result in generating Pareto front.

As the problem is NP-Hard, the exact method, i.e., epsilon-constraint, cannot solve the large size instances. So, after justifying the efficiency of proposed NSGA-II for small and medium size instances, the NSGA-II is used to find the Pareto front of large size instances.

In order to compare them, the best result obtained from the both methods is considered. In Table 2 the results of small and medium instances are presented.

Table 2: The results of calculations for small and medium sizes

Prob. No.	ε-constraint			
	f_1	Time(s)	f_2	Time(s)
04/02/02	6008	1	40	12
06/02/03	6639	55	56	623
08/02/03	6985	72	85	796
08/03/03	9544	94	72	1083
09/03/04	9863	356	122	1318
10/03/03	9759	271	80	1229
10/03/05	10146	1018	94	2477
12/03/05	14215	3293	109	5145
12/04/04	16489	3124	30	5823
14/04/05	17946	5894	112	9857
Mean	**10759.4**	**1417.8**	**80**	**2836.3**

Prob. No.	NSGA-II			
	f_1	Time(s)	f_2	Time(s)
04/02/02	6008	1	40	1
06/02/03	6639	4	56	5
08/02/03	6989	5	86	7
08/03/03	9563	6	72	10
09/03/04	9980	14	125	19
10/03/03	9822	14	81	18
10/03/05	10358	27	96	34
12/03/05	14362	29	110	46
12/04/04	16576	38	30	48
14/04/05	18356	63	115	71
Mean	**10865.3**	**20.1**	**81.1**	**25.9**

Prob. No.	Error	
	f_1	f_2
04/02/02	0.0%	0.0%
06/02/03	0.0%	0.0%
08/02/03	0.06%	1.16%
08/03/03	0.19%	0.0%
09/03/04	1.17%	2.4%
10/03/03	0.64%	1.23%
10/03/05	2.04%	2.08%
12/03/05	1.02%	0.9%
12/04/04	0.52%	0.0%
14/04/05	2.13%	2.6%
04/02/02	0.0%	0.0%
Mean	**0.649713**	**1.037**

In Table 2, the first column shows the number of the problem. Each number includes three parts. From left to right, they show the number of customers, the number of potential warehouses, and the number of transportation vehicles, respectively. The second to fifth columns shows the objective values and CPU times for epsilon-constraint method, while, in the same way, in sixth to ninth columns, the values related to NSGA-II have been presented. In the last two columns, the errors of NSGA-II in comparison with result of epsilon-constraint method have been shown for each objective function.

A review of the CPU time for NSGA-II algorithm and epsilon-constraint method shows that as the size of the problem is increased, the time needed for the solution by epsilon-constraint method is dramatically increased, while the increase of CPU time for NSGA-II algorithm is approximately slow. As the number of constraints of epsilon-constraint method is highly dependent on the number of transportation vehicles, so the increase in the number of transportation vehicles, compared to other factors, leads to the increase in the cost objective. When the number of distribution centers is increased, the required CPU time for obtaining the second objective function is increased, because it is dependent on the number of distribution centers. The levels of objective function error in all small and medium size instances is less than %3, which demonstrates the efficiency and reliability of the proposed NSGA-II algorithm. So, we may trust on the result of the proposed NSGA-II for large size instances.

8.2 Comparison Metrics

In order to evaluate the accuracy and diversity of regenerated Pareto front by both methods on several instances, comparison metrics are used. In this study two metrics, called mean ideal distance, and spacing metric are used.

8.3 Mean Ideal Distance (MID)

The value of MID is equal to the distance between solutions on Pareto front of the NSGA-II algorithm and the ideal point. In this study, we set the ideal point as the zero. The MID can be calculated using (29).

$$MID = \frac{\sum_{i=1}^{n} \sqrt{\left(\frac{f_{1i} - f_1^{best}}{f_{1,total}^{max} - f_{1,total}^{min}}\right)^2 + \left(\frac{f_{2i} - f_2^{best}}{f_{2,total}^{max} - f_{2,total}^{min}}\right)^2}}{n} \tag{29}$$

Where, n is the number generated Pareto solutions, and$f_{i,total}^{max}$ and$f_{i,total}^{min}$, $i = 1,2$ are the achieved maximum and minimum values of objective functions. The coordinates of ideal points is (f_2^{best}, f_1^{best}) in (29). As both objective functions are to be minimized, it is set equal to (0,0). The low value of MID indicates that the algorithm has a higher quality. The MID checks the accuracy of re-generated Pareto front.

Spacing Metric (SM)

SM shows the flatness of Pareto front distribution. This metric is calculated using (30).

$$SM = \frac{\sum_{i=1}^{n-1} \left|\bar{d} - d_i\right|}{(n-1)\bar{d}} \tag{30}$$

Where d_i is the Euclidean distance between the two adjacent solutions of Pareto Front. Also, $\bar{d}$is the mean of d_i, *i=1,…, n*. The lower SM measurement shows fair dispersion on Pareto front. On the other hand in this condition, all distances between two consecutive answers have been fairly distributed. If the SM is near to zero, all distances between two consecutive answers on Pareto front are equal. The SM checks the diversity of re-generated Pareto front.

8.4 The results of Comparison

As mentioned, ten small and medium size test problems presented in Table 2, are solved by proposed NSGA-II and epsilon-constraint method. The, regenerated Pareto solutions are compared MID, and SM. The results of this comparison are presented in Table 3.

Table 3: Results of numerical solution for small and medium sizes

R	Problem number	MID		SM	
		NSGA-II	ε-constraint	NSGA-II	ε-constraint
1	04/02/02	7.42	7.31	0.82	0.11
2	06/02/03	7.01	6.95	1.12	0.15
3	08/02/03	8.11	7.81	0.89	0.09
4	08/03/03	7.79	7.72	1.65	0.21
5	09/03/04	7.48	7.05	1.38	0.39
6	10/03/03	7.52	7.35	0.79	0.12
7	10/03/05	7.57	7.42	1.21	0.31
8	12/03/05	7.71	7.28	1.53	0.25
9	12/04/04	7.43	7.15	1.15	0.14
10	14/04/05	7.80	7.51	0.84	0.08
Mean		**7.58**	**7.36**	**1.14**	**0.19**

It can be concluded from Table 3 that, the average value of MID for all 10 instance is equal to 7.36 for epsilon-constraint method. This value is 7.58 for NSGA-II algorithm. It shows that the mean of distances between Pareto answers and the ideal answer is slightly better in epsilon-constraint method. In other words, the Pareto answers obtained by the proposed NSGA-II algorithm, compared to the obtained answers by epsilon constraint method, were %2.9 further away from ideal answer (coordinates basis) on average. Although, we cannot conclude the overall performance of proposed NSGA-II algorithm regardless to SM metric and CPU time. The value of SM metric shows that the dispersion of answers obtained by NSGA-II algorithm is equal to 1.14 on average. This value is equal to 0.19 for the answers obtained by epsilon-constraint method. It can be concluded that both MID, and SM are slightly better in epsilon-constraint method for average results on 10 small and medium size instances. Although the average CPU time is 20 times bigger in epsilon-constraint method. This is direct result of size of the instances. As the problem is NP-Hard the epsilon-constraint method is not able to achieve a solution in a polynomial time manner.

In order to compare the proposed algorithm with epsilon-constraint method, the regenerated Pareto front of both methods is shown in Figure 8.

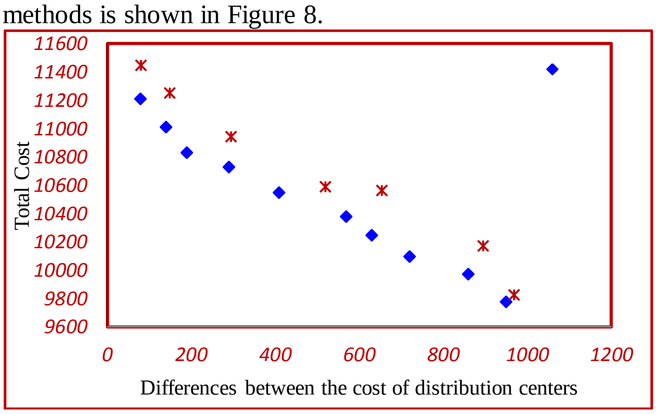

Figure 8: Re-generated Pareto Front on Small and Medium Instances

Based on these results, the indices in this study show that the proposed NSGA-II algorithm is efficient and reliable enough to handle large size test and real world problem which are not solvable using epsilon-constraint. Now we are confident that the algorithm is efficient enough in small and medium size instances. So, the Pareto front achieved by proposed NSGA-II is reliable enough to be interpreted and used. Ten large size instances are designed and solved by proposed NSGA-II algorithm. Then, the obtained Pareto front evaluated by the metrics. The results are shown in Table 4.

Table 4: Results of NSGA-II algorithm on large size test problems

Problem number	MID	SM	Time (s)
18/05/05	7.23	1.22	104
20/05/06	7.44	0.83	351
25/05/07	7.67	1.74	1289
30/06/07	7.98	0.89	1927
40/07/09	8.21	1.27	3745
50/08/10	7.24	1.94	4256
60/09/11	8.02	1.14	6282
70/10/12	7.25	0.71	8045
80/11/14	7.17	1.62	10621
100/12/15	7.95	0.75	14548
Mean	**7.61**	**1021**	**5116/8**

The results in Table 4 show that the algorithm has acceptable performance for large size test problems. Figure 9 presents the generated Pareto front for a large size instance including 80 customers, 11 potential distribution center, and 14 transportation vehicles using proposed NSGA-II method. It is notable that none of the large size test problems in Table 4 are solvable using epsilon-constraint method.

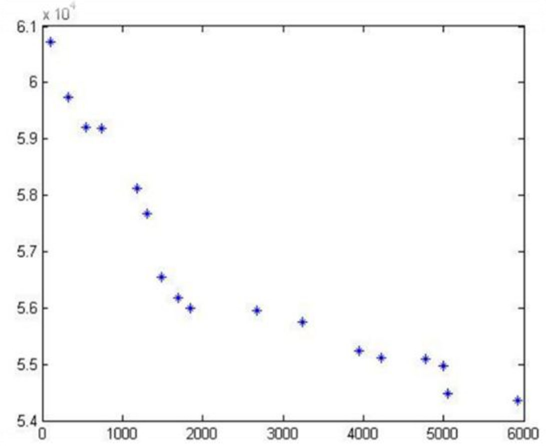

Figure 9: Pareto Front of Large Size Test Problem

9.Conclusion Future Research Directions

In this paper, a bi-objective location-routing mathematical programming was proposed to distribute perishable products in supply chains. The objectives were to minimize the total cost of the system and to balance the transportation cost of perishable products. Because these items are perishable, on time delivery is an essential factor in distribution process. In comparison with other existing models which only consider cost of distribution, the proposed model of this paper also consider workload balance of distribution centers. Two solution procedures, i.e. an exact method called, epsilon-constraint and an evolutionary computation, called NSGA-II algorithm, were proposed to generate non-dominated solution on Pareto front of instances of the problem. The performances of both methods were compared on several benchmark instances using accuracy and diversity metrics. The acceptable performance of proposed NSGA-II was demonstrated through several runs. Then, the proposed NSGA-II was used to solve large sizes instances. The Pareto front of large size instances were achieved successfully using proposed NSGA-II.

Further researches in this area can be accomplished. Other objective functions, such as balancing the number of customers which are served by each distribution center, balancing the time that transportation vehicles are operating, can be modeled in future studies. Extra assumptions such as, heterogeneous transportation vehicles with different capacities, speeds, and cost, one-way routes, traffic factor throughout the routes, fuel shares, drivers' level of skill and wages, and also accidents, failure of vehicles are among the points which might bring the subject of these studies closer to the real life cases.

References

[1] Ahmadi-Javid, A., Seddighi, A.-H. 2013. A location-routing problem with disruption risk, Transportation Research Part E: Logistics and Transportation Review 53 63-82.

[2] Alumur, S., Kara, B.Y. 2007. A new model for the hazardous waste location-routing problem, Computers & Operations Research 34, 1406-1423.

[3] Amiri, N.,Tavakkoli-Moghaddam, R., Gholipour-Kanani, Y., Toarbi, S.A. 2012. Modelling a Novel Multi-Objective Open-Shop Scheduling Problem and Solving by a Scatter Search Method, International Journal of Industrial Engineering & Production Management 23 149-160.

[4] Banos, R., Ortega, J., 2013. A hybrid meta-heuristic for multi-objective vehicle routing problems with time windows , Computers & Industrial Engineering 65 286–296.

[5] Bramel, J., Simchi-Levi, D., 1997. The logic of logistics: Theory, Algorithms, and Applications for Logistics Management. Springer.

[6] Branke, J., Deb, K., Miettinen, K., Slowinski, R. 2008. Multi objective Optimization: Interactive and Evolutionary Approaches, Theoretical computer science, Springer:.

[7] Christofides, N. Elion, S. 1969. Algorithm for the vehicle-dispatching problem, Operational Research Quarterly 3 309-318.

[8] Clarke, G., Wright, J.W. 1964. Scheduling of vehicles from a central depot to a number of delivery points, Operations Research, 12 568–589.

[9] Dantzig, G., Ramser, J.H. 1959. The truck dispatching problem, Management Science 6 80-91.

[10] Deb, K., Paratap, A., Sammeer, A., Meyarivan, T. 2002. A fast elitist multiobjective genetic algorithm: NSGA-II, IEEE Transactions on Evolutionary computation 6 182-197.

[11] Fisher, M. 1995. "Vehicle routing", Handbooks in OR and MS, M.O. Ball et al., Vol. 8.

[12] Mavrotas, G. 2009. Effective implementation of the ε-constraint method in Multi-Objective Mathematical Programming problems, Applied Mathematics and Computations 213 455–465.

[13] Meisel, F., Kirschstein, T., Bierwirth Ch. 2013. Integrated production and intermodal transportation planning in large scale production–distribution-networks, Transportation Research Part E: Logistics and Transportation Review 60 62-78.

[14] Mohammadi, M., Torabi, S.A., Tavakkoli-Moghaddam R. 2014. Sustainable hub location under mixed uncertainty, Transportation Research Part E: Logistics and Transportation Review 62 89-115.

[15] Nagy. G., Salhi, S. 2007. Location- routing: Issues, models and methods, European Journal of Operational Research 176 649-672.

[16] Najafi, M., Eshghi, K., Dullaert, W. 2013. A multi-objective robust optimization model for logistics planning in the earthquake response phase, Transportation Research Part E: Logistics and Transportation Review 49 217-249.

[17] Potvin, J.Y., Gendreau, M., 2009. An exact ε-constraint method for bi-objective combinatorial optimization problems: Application to the Traveling Salesman Problem with Profits, European Journal of Operational Research 194 39–50.

[18] Reimann, M., Stummer, M. and Doerner, K. 2002. "A savings based ant system for the vehicle routing problem", W.B. Langdon, et al. (Eds.). Kaufmann M (GECCO 2002) Proceedings of the Genetic and Evolutionary Computation Conference, San Francisco.

[19] Salhi, S. and Rand, G.K, 1989. The effect of ignoring routes when locating depots. European Journal of Operational Research 39 150-156.

[20] Webb, M.H.J. 1968. Cost functions in the location of depots for multiple-delivery journeys. Operational Research Quarterly 19 311–320.